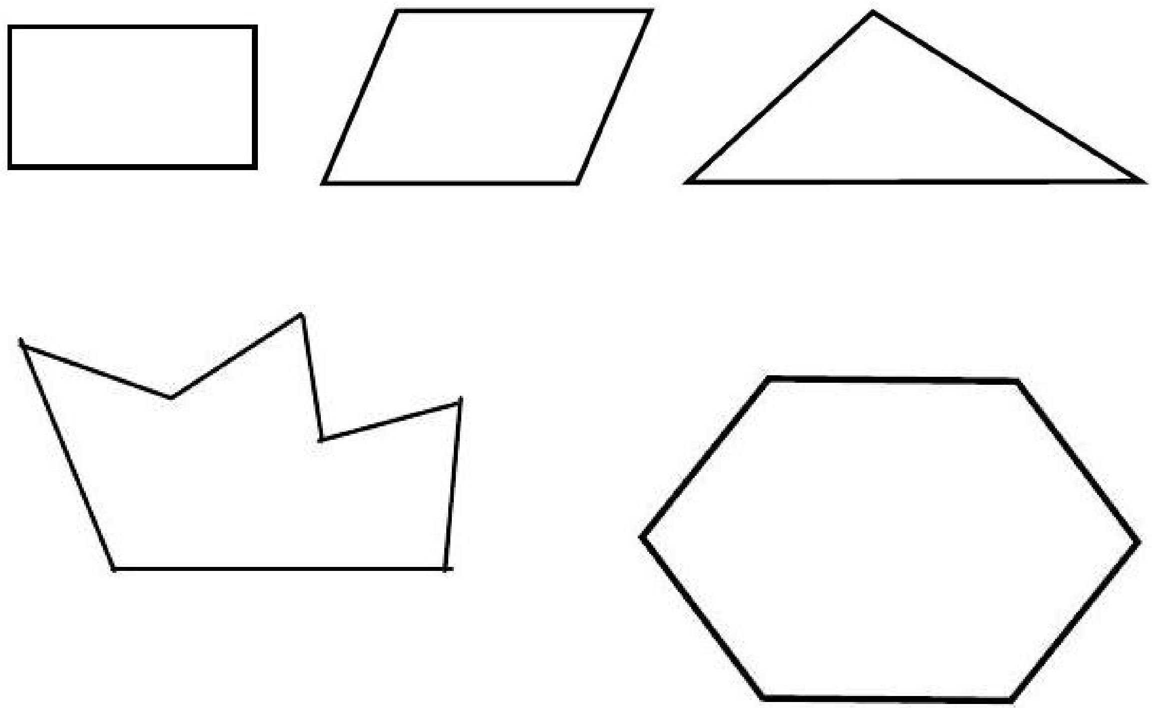
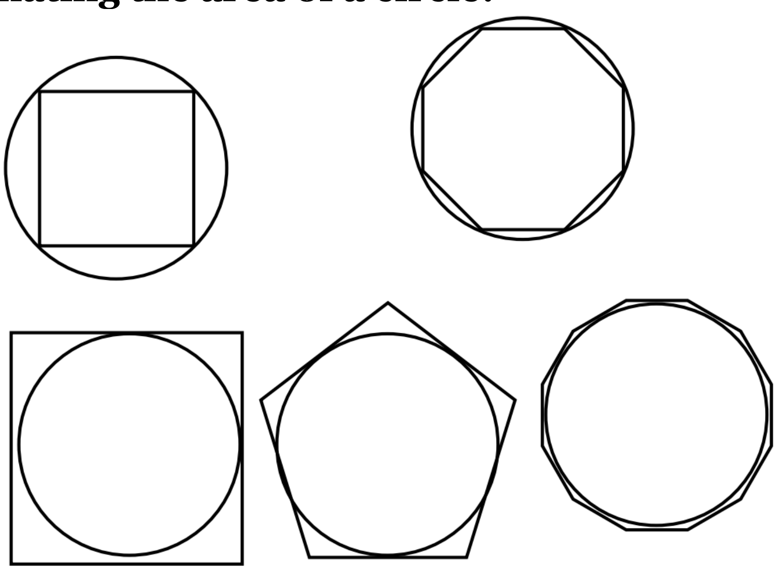
Math 1210 #2
Introduction to Area

Area of a Polygon:



Estimating the area of a circle:



Sums and Sigma Notation

$1 + 2 + 3 + 4 + \cdots + 100 =$

$2 + 4 + 6 + 8 + \cdots + 1000 =$

$1 + 4 + 9 + 16 + \cdots + 625 =$

Σ = Sigma, the capitol Greek letter called "sigma";

It means summation. i = index

$$\sum_{j=1}^{n} \frac{1}{j} =$$

$$\sum_{i=1}^{n} c =$$

Linearity of Σ

Let $\{a_i\}$ and $\{b_i\}$ denote two sequences and c is a real number.

i. $\sum_{i=1}^{n} c\, a_i = c \sum_{i=1}^{n} a_i$

ii. $\sum_{i=1}^{n} a_i \pm b_i = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

Special Sum Formulas

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{i=1}^{n} i^4 = 1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(6n^3 + 9n^2 + n - 1)}{30}$$

EX 1

$$\sum_{i=1}^{10}[(i-1)(4i+3)]$$

EX 2

$$\sum_{j=1}^{n}(2\mathrm{j}-3)^2$$

EX 5

Change the variable in the index to start at 1.

$$\sum_{k=5}^{14} k2^{k-4}$$

Collapsing Sum

$$\sum_{i=1}^{n}(a_{i+1}-a_i) = a_{n+1}-a_1$$

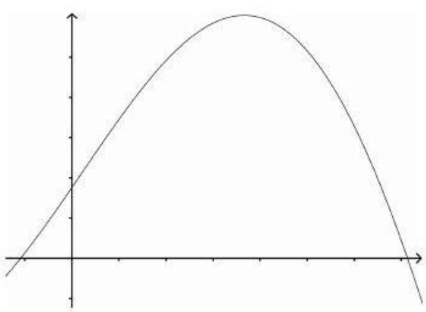
EX 3

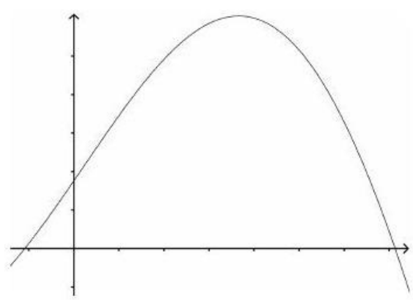
$$\sum_{k=1}^{10}\left(2^k-2^{k-1}\right)$$

EX 4

$$\sum_{k=3}^{m+1} (a_k - a_{k-1})$$

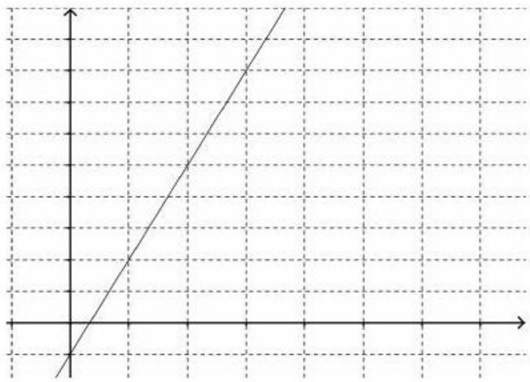
We will estimate the area under a curve using inscribed or circumscribed rectangles.

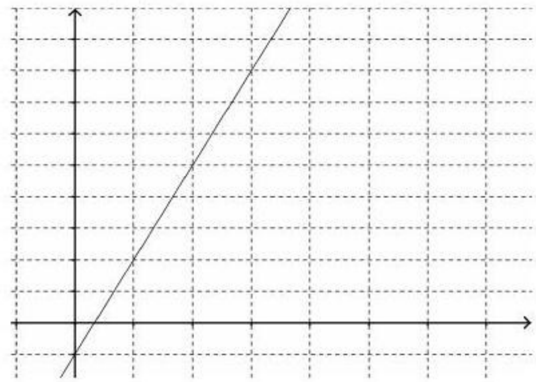


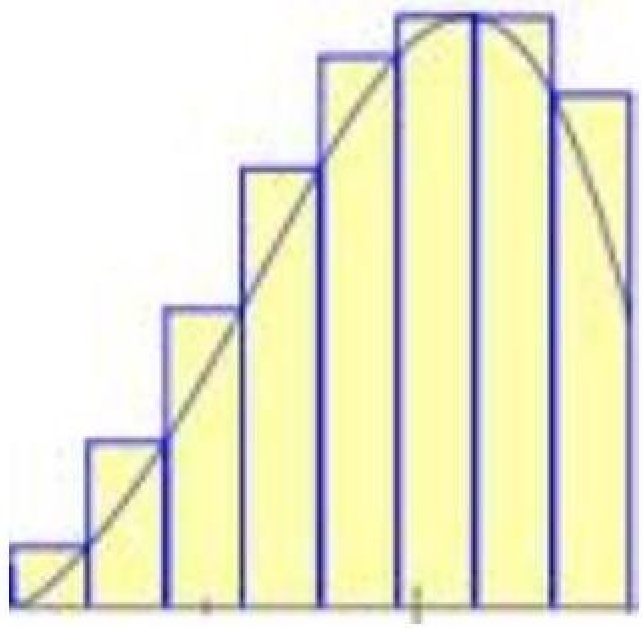


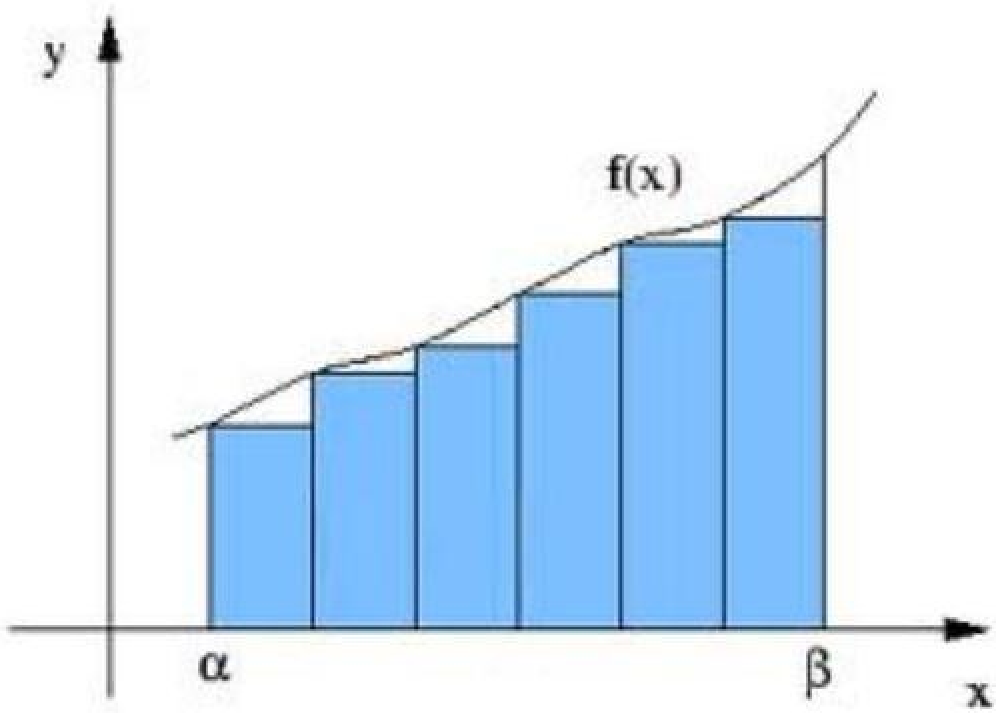
EX 6

For $f(x) = 3x - 1$, divide the interval [1,3] into 4 equal subintervals.
Calculate the area of the circumscribed rectangles.








y
f(x)
α
β
x