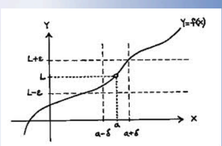
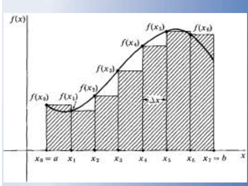


$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$



$$\lim_{max\,\Delta x_i \to 0} \sum_1^n f(x_i)\Delta x_i = \int_a^b f(x)dx$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

The Second Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

$$\int_a^b f'(x)dx = f(b) - f(a)$$

Second Fundamental Theorem of Calculus

Let f be continuous on $[a,b]$ and F be any antiderivative of f on $[a,b]$.

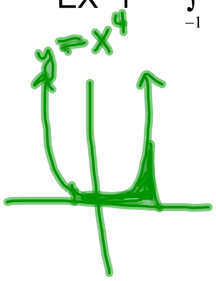
Then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

note: F(upper bound) − F(lower bound)

EX 1 $\int_{-1}^{2} x^4\,dx$

$$= \frac{x^5}{5}\Big|_{-1}^{2}$$



$$= \frac{2^5}{5} - \frac{(-1)^5}{5} = \frac{32}{5} - \frac{-1}{5}$$

$$= \frac{32}{5} + \frac{1}{5} = \frac{33}{5}$$

EX 2 $\int_{\pi/6}^{\pi/2} 2\sin t\,dt$

$$= 2\int_{\pi/6}^{\pi/2} \sin t\,dt$$

$$= 2(-\cos t)\Big|_{\pi/6}^{\pi/2}$$

$$= 2\left(-\cos(\pi/2) - -\cos(\pi/6)\right)$$

$$= 2\left(0 + \sqrt{3}/2\right)$$

$$= \sqrt{3}$$

Substitution Rule for Indefinite Integrals

Let *g* be differentiable and *F* be any antiderivative of *f*.
Then if *u* = *g(x)*,

(u- substitution)

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du = F(u) + C = F(g(x)) + C$$

("undoing" chain rule of differentiation)

EX 3 $\int \sqrt{x^3+1}\,\underbrace{(3x^2)\,dx} = \int \sqrt{u}\;du$

$u = x^3 + 1$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$= \int u^{1/2}\,du$

$= \frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{3}(x^3+1)^{3/2} + C}$

EX 4 $\int_0^{\pi/2} \sin^2(3x)\cos(3x)\,dx$ (definite integral)

① $u = \cos(3x)$

$\frac{du}{dx} = -3\sin(3x)$

$du = -3\sin(3x)\,dx$

$-\frac{1}{3}du = \sin(3x)\,dx$

won't work!!

$\int \sin^2(3x)\cos(3x)\,dx$

$= \int \underbrace{\cos(3x)}_{u}\underbrace{\sin(3x)}_{?}\underbrace{(\sin(3x)\,dx)}_{-\frac{1}{3}du}$

② $u = \sin(3x)$

$du = 3\cos(3x)\,dx$

$\frac{1}{3}du = \cos(3x)\,dx$

$\int_0^{\pi/2} \sin^2(3x)\cos(3x)\,dx$

$= \int_0^{-1} u^2\left(\frac{1}{3}\right)du$

convert limits of integration:

$u = \sin(3x)$

$x = 0,\; u = \sin(3\cdot 0) = 0$

$x = \pi/2,\; u = \sin\left(\frac{3\pi}{2}\right) = -1$

$= \frac{1}{3}\int_0^{-1} u^2\,du = \frac{1}{3}\left(\frac{u^3}{3}\Big|_0^{-1}\right)$

(don't go back in terms of x)

$= \frac{1}{9}\left((-1)^3 - 0^3\right)$

$= \frac{1}{9}(-1) = \boxed{-1/9}$

27B Second Fundamental Thm

EX 5 $\int_1^3 \frac{x^2+1}{\sqrt{x^3+3x}}dx$

$u = x^3+3x$

$\frac{du}{dx} = 3x^2+3$

$du = 3(x^2+1)dx$

$\frac{1}{3}du = (x^2+1)dx$

$x=1,\ u=1^3+3(1)=4$

$x=3,\ u=3^3+3(3)=36$

$= \int_4^{36} \frac{1/3}{\sqrt{u}}du$

$= \frac{1}{3}\int_4^{36} u^{-1/2}\,du$

$= \frac{1}{3}\left(2u^{1/2}\Big|_4^{36}\right)$

$= \frac{2}{3}\left(\sqrt{u}\Big|_4^{36}\right)$

$= \frac{2}{3}\left(\sqrt{36}-\sqrt{4}\right) = \frac{2}{3}(6-2)$

$= \boxed{\frac{8}{3}}$

EX 6 $\int_{-4}^{-1} \frac{1-s^4}{2s^2}ds$

$= \int_{-4}^{-1} \frac{1}{2s^2} - \frac{s^4}{2s^2}\,ds$

$= \int_{-4}^{-1} \frac{1}{2}s^{-2} - \frac{1}{2}s^2\,ds$

$= \left(\frac{1}{2}\left(\frac{s^{-1}}{-1}\right) - \frac{1}{2}\left(\frac{s^3}{3}\right)\right)\Big|_{-4}^{-1}$

$= \left(\frac{-1}{2s} - \frac{s^3}{6}\right)\Big|_{-4}^{-1}$

$= \left(\frac{-1}{-2} - \frac{-1}{6}\right) - \left(\frac{-1}{2(-4)} - \frac{(-4)^3}{6}\right)$

$= \frac{1}{2} + \frac{1}{6} - \frac{1}{8} - \frac{64}{6}$

$= \frac{3}{8} - \frac{63}{6} = \frac{3}{8} - \frac{21}{2}\left(\frac{4}{4}\right)$

$= \frac{3}{8} - \frac{84}{8}$

$= \boxed{-\frac{81}{8}}$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$