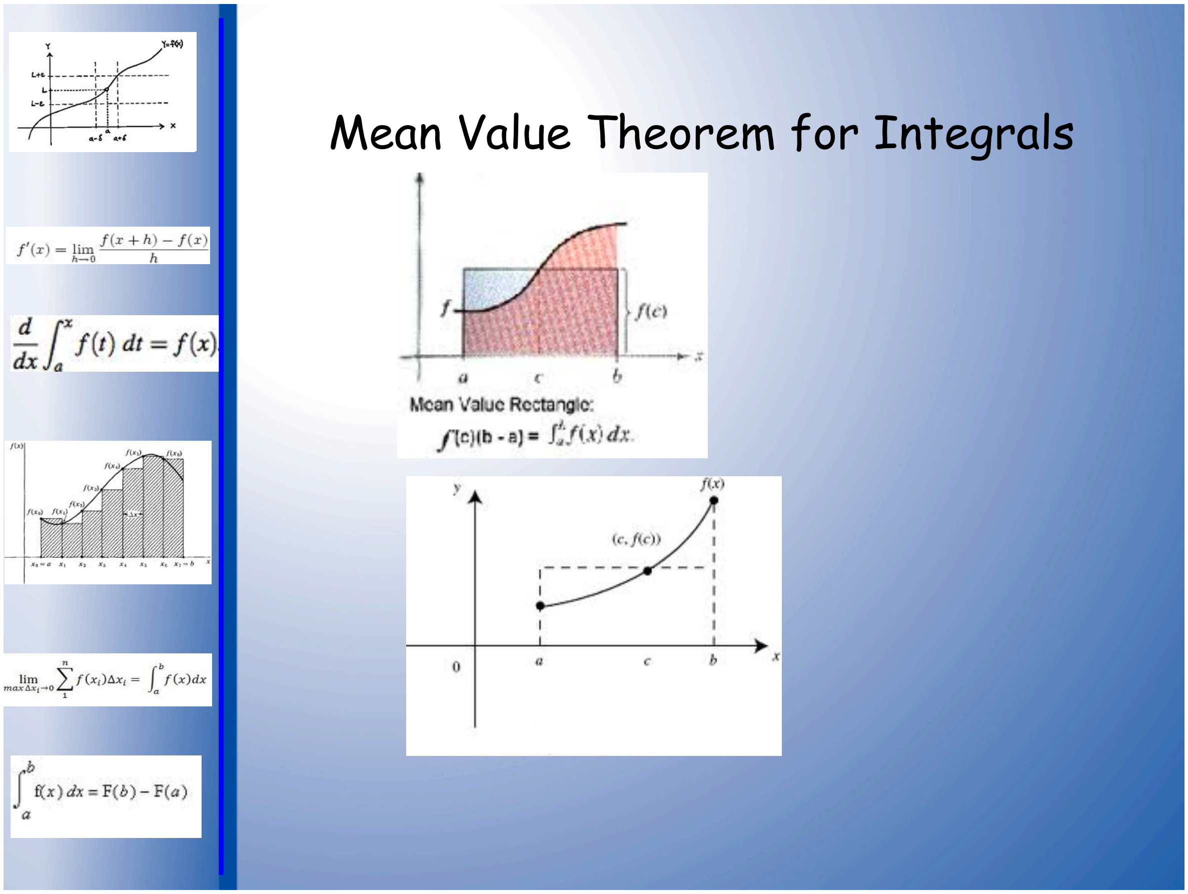
28 MVT Integrals



Definition Average Value of a Function

If *f* is integrable on [*a,b*], then the average value of *f* on [*a,b*] is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

EX 1   Find the average value of this function on [0,3] $f(x) = \frac{x}{\sqrt{x^2+16}}$

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Mean Value Theorem for Integrals

If *f* is continuous on [*a,b*] there exists a value *c* on the interval (*a,b*) such that

$$\int_a^b f(t)\,dt = f(c)(b-a) \quad .$$

EX 2 Find the values of *c* that satisfy the MVT for integrals on [*0,1*].

$f(x) = x(1-x)$

EX 3 Find values of c that satisfy the MVT for integrals on $[3\pi/4\ ,\ \pi]$.

$f(x)=cos(2x-\pi)$

Symmetry Theorem

If *f* is an even function, then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

If *f* is an odd function, then $\int_{-a}^{a} f(x)\,dx = 0$.

Theorem

If *f* is a periodic function with period *p*, then $\int_{a+p}^{b+p} f(x)\,dx = \int_{a}^{b} f(x)\,dx$.

EX 4 $\int_{-\pi/2}^{\pi/2} x^2 \sin^2\left(x^3\right)\cos\left(x^3\right) dx$

EX 5 $\int_{-\pi/2}^{\pi/2} x \sin^2\left(x^3\right)\cos\left(x^3\right) dx$

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