Math 1210 #28A
Mean Value Theorem for Integrals

Definition: Average Value of a Function

If f is integrable on $[a, b]$, then the average value of f on $[a, b]$ is

$$\frac{1}{b-a}\int_a^b f(x)\text{dx}$$

EX 1

Find the average value of this function on $[0,3]$ $f(x) = \frac{x}{\sqrt{x^2+16}}$

Mean Value Theorem for Integrals

If f is continuous on $[a, b]$ there exists a value c on the interval (a, b) such that

$$\int_a^b f(t)dt = \mathrm{f}(c)(b - a).$$

EX 2

Find the values of c that satisfy the MVT for integrals on [0,1].

$$\mathrm{f}(x) = \mathrm{x}(1 - x)$$

EX 3

Find values of c that satisfy the MVT for integrals on $[3\pi/4, \pi]$.

$$\mathrm{f}(x) = \cos(2x - \pi)$$

Symmetry Theorem

If f is an even function, then $\int_{-a}^{a} f(x)\mathrm{dx} = 2\int_{0}^{a} f(x)\mathrm{dx}$.

If f is an odd function, then $\int_{-a}^{a} f(x)\mathrm{dx} = 0$.

Theorem

If f is a periodic function with period p, then $\int_{a+p}^{b+p} \mathrm{f}(x)\mathrm{dx} = \int_{a}^{b} \mathrm{f}(x)\mathrm{dx}$.

EX 4

$$\int_{-\pi/2}^{\pi/2} x^2 \sin^2(x^3)\cos(x^3)\mathrm{dx}$$

EX 5

$$\int_{-\pi/2}^{\pi/2} \mathrm{x}\, sin^2(x^3)cos(x^3)dx$$