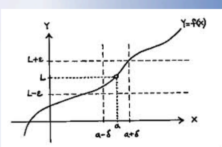
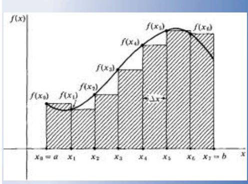
Area of a Plane Region



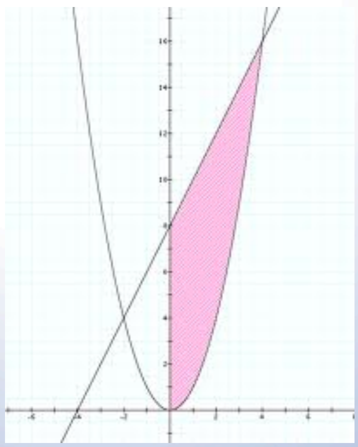
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

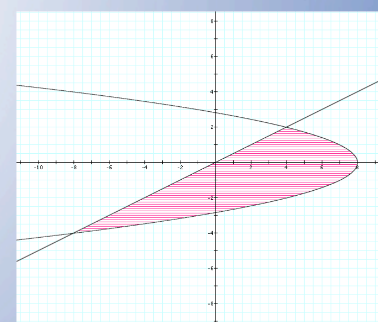
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$



$$\lim_{\max \Delta x_i \to 0} \sum_{1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$



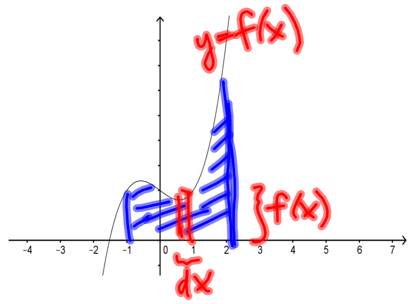


29B Area of Plane Region

A = The area between a curve, *f(x),* and the *x-axis* from *x=a* to *x=b* is found by

$$\int_a^b f(x)\,dx .$$

EX 1 Find the area of the region between the function and the *x-axis* on the *x*-interval [*-1,2*]. $f(x) = x^3 - x + 2$



$$\int_{-1}^{2} f(x)\,dx = \int_{-1}^{2} (x^3 - x + 2)\,dx$$

$$= \left(\frac{x^4}{4} - \frac{x^2}{2} + 2x\right)\Big|_{1}^{2}$$

$$= \left(\frac{2^4}{4} - \frac{2^2}{2} + 2(2)\right) - \left(\frac{(-1)^4}{4} - \frac{(-1)^2}{2} + 2(-1)\right)$$

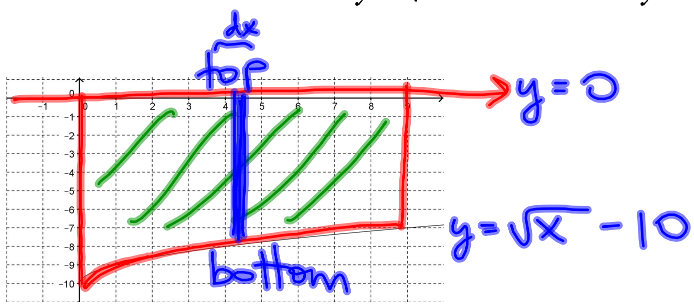
$$= (4 - 2 + 4) - \left(\frac{1}{4} - \frac{1}{2} - 2\right)$$

$$= 6 + 2\tfrac{1}{4}$$

$$= \boxed{8\tfrac{1}{4}}$$

EX 2 Find the area between $y=\sqrt{x}-10$ and $y=0$ between $x=0$ and $x=9$.

x-axis



$$A=\int_a^b (\text{ht of rectangle})(\text{width rect})$$

$$=\int_0^9 \left(0-(\sqrt{x}-10)\right)\,dx$$

$$=\int_0^9 -\sqrt{x}+10\,dx$$

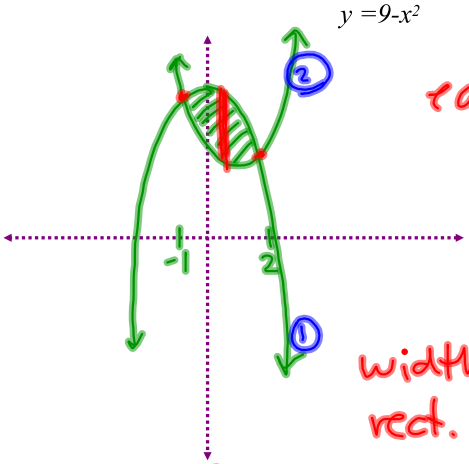
$$=\left(-\frac{2}{3}x^{3/2}+10x\right)\Big|_0^9$$

$$=\left(-\frac{2}{3}(9^{3/2})+10(9)\right)-\cancel{\left(-\frac{2}{3}(0)+10(0)\right)}^{\,0}$$

$$=-2(9)+90=\boxed{72}$$

EX 3 Find the area between these two curves.

① $y = 9 - x^2$ ② $2y = x^2 - 3x + 12 \Leftrightarrow y = \frac{1}{2}x^2 - \frac{3}{2}x + 6$



each sample rect. bndd above by ① + below by ②

$\Rightarrow$ ht of rect $= (9 - x^2) - (\frac{1}{2}x^2 - \frac{3}{2}x + 6)$

$= -\frac{3}{2}x^2 + \frac{3}{2}x + 3$

width of each rect. = dx

$A = \int_{?}^{?} (-\frac{3}{2}x^2 + \frac{3}{2}x + 3)\,dx$

(need the pts of intersection)

$y = 9 - x^2, \quad y = \frac{1}{2}x^2 - \frac{3}{2}x + 6$

$9 - x^2 = \frac{1}{2}x^2 - \frac{3}{2}x + 6$

$\frac{2}{3}(0) = (\frac{3}{2}x^2 - \frac{3}{2}x - 3)\frac{2}{3}$

$0 = x^2 - x - 2$

$0 = (x - 2)(x + 1)$

$x = 2, -1$

$= \int_{-1}^{2} (-\frac{3}{2}x^2 + \frac{3}{2}x + 3)\,dx$

$= \left(-\frac{3}{2}\left(\frac{x^3}{3}\right) + \frac{3}{2}\left(\frac{x^2}{2}\right) + 3x\right)\Big|_{-1}^{2}$

$= \left(-\frac{1}{2}x^3 + \frac{3}{4}x^2 + 3x\right)\Big|_{1}^{2}$

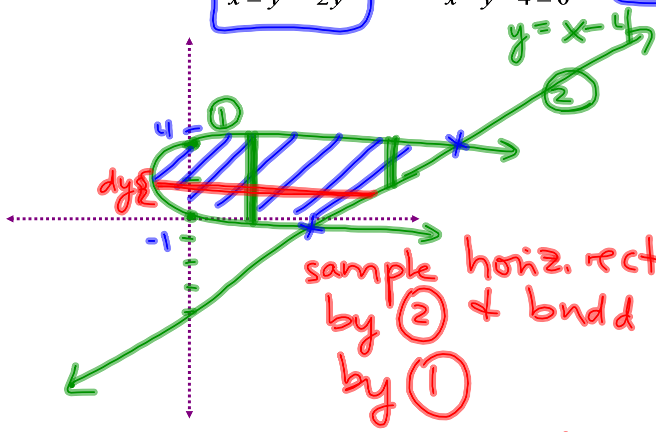
$= \left(-\frac{1}{2}(8) + \frac{3}{4}(4) + 6\right) - \left(\frac{1}{2} + \frac{3}{4} - 3\right)$

$= (-4 + 3 + 6) - (1\frac{1}{4} - 3)$

$= 5 - (-1\,3/4) = \boxed{6\frac{3}{4} \text{ or } \frac{27}{4}}$ units^2

EX 4 Find the area of the region bounded by these two curves.

① $\boxed{x = y^2 - 2y}$ ② $x - y - 4 = 0$ $\boxed{x = y + 4}$



sample horiz. rect: bndd right ("top") by ② & bndd on left ("bottom") by ①

width of rect = dy

length/ht. of rect = horiz dist from ① to ②

(dist ② − ①)

$$A = \int_{-1}^{4} \left(y + 4 - (y^2 - 2y)\right) dy$$

$$= \int_{-1}^{4} \left(-y^2 + 3y + 4\right) dy$$

$$= \left(\frac{-y^3}{3} + \frac{3y^2}{2} + 4y\right)\Big|_{-1}^{4}$$

$$= \left(\frac{-64}{3} + 3(8) + 16\right) - \left(\frac{1}{3} + \frac{3}{2} - 4\right)$$

$$= \frac{-65}{3} + 40 + 4 - \frac{3}{2}$$

$$= 44 - \frac{65}{3}\left(\frac{2}{2}\right) - \frac{3}{2}\left(\frac{3}{3}\right)$$

$$= 44 - \frac{130}{6} - \frac{9}{6}$$

$$= 44 - \frac{139}{6} = \frac{44(6) - 139}{6}$$

$$= \frac{264 - 139}{6} = \boxed{\frac{125}{6}}$$

pts of intersectn

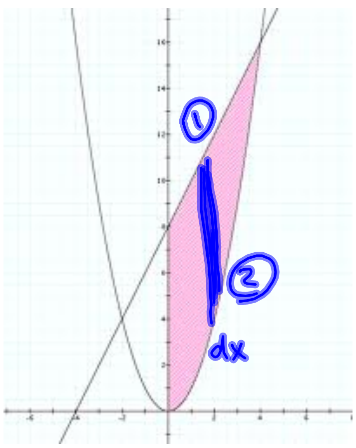
$x = y + 4$, $x = y^2 - 2y$

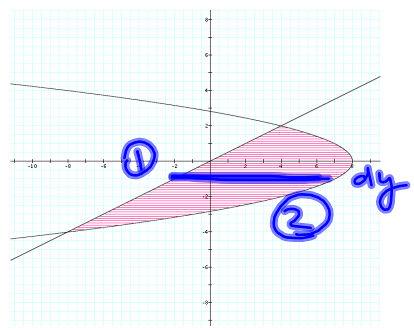
$y + 4 = y^2 - 2y$

$0 = y^2 - 3y - 4$

$0 = (y + 1)(y - 4)$

$y = -1, 4$


1
2
dx


1
2
dy