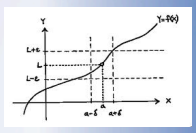
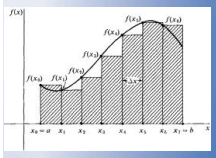
31 Length Curve



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

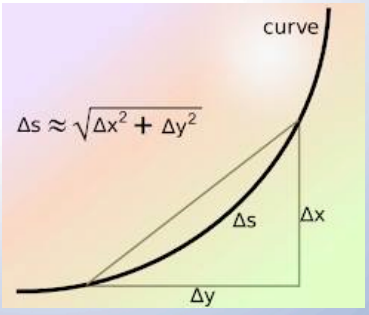
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

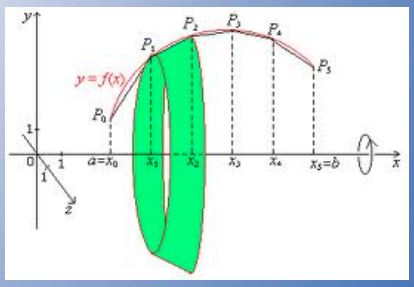


$$\lim_{\max \Delta x_i \to 0} \sum_1^n f(x_i)\Delta x_i = \int_a^b f(x)dx$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Length of a Plane Curve and Surface Area





Length of a Plane Curve

A plane curve is a curve that lies in a two-dimensional plane. We can define a plane curve using parametric equations. This means we define both x and y as functions of a parameter.

Parametric equations

Definition

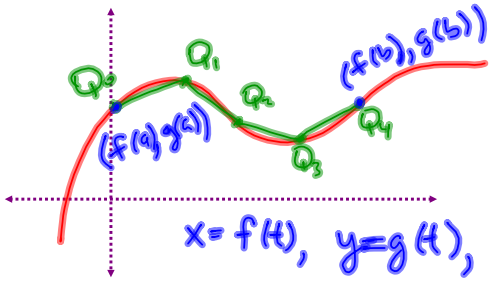
A plane curve is smooth if it is given by a pair of parametric equations $x = f(t)$, and $y = g(t)$, t is on the interval $[a,b]$ where f' and g' exist and are continuous on $[a,b]$ and $f'(t)$ and $g'(t)$ are not simultaneously zero on (a,b).

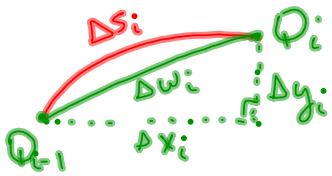
31 Length Curve

EX 1 Sketch the graph of the curve given by these parametric equations.
$x = 3t^2+2 \quad y = 2t^2-1 \quad 1 \le t \le 4$

Arc length

We can approximate the length of a plane curve by adding up lengths of linear segments between points on the curve.

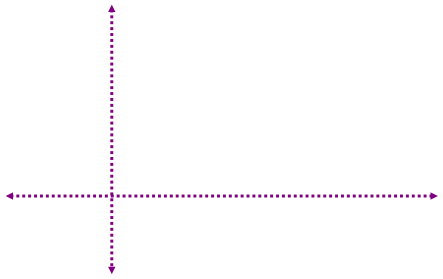




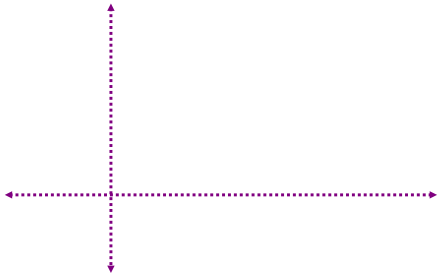
$$L = \int_a^b \sqrt{\left[f'(t)\right]^2 + \left[g'(t)\right]^2}\, dt = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

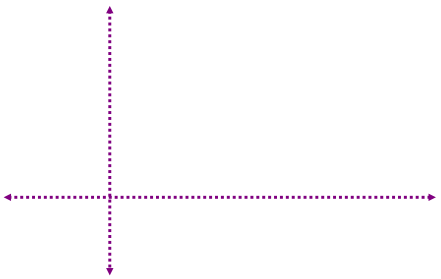
EX 2 Find the circumference of the circle $x^2 + y^2 = r^2$.



EX 3 Find the length of the line segment on $2y - 2x + 3 = 0$ between $y = 1$ and $y = 3$.
Check your answer using the distance formula.



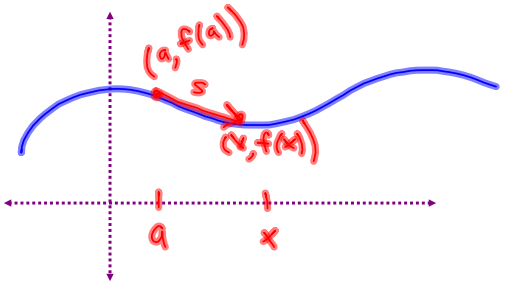
EX 4 Find the arc length of the curve $f(x) = \sqrt{x}$ from $x = 0$ to $x = 4$.



Surface Area

Differential of Arc Length

Let f(x) be continuously differentiable on [a,b]. Start measuring arc length from (a,f(a)) up to (x,f(x)), where a is a real number. Then, the arc length is a function of x.



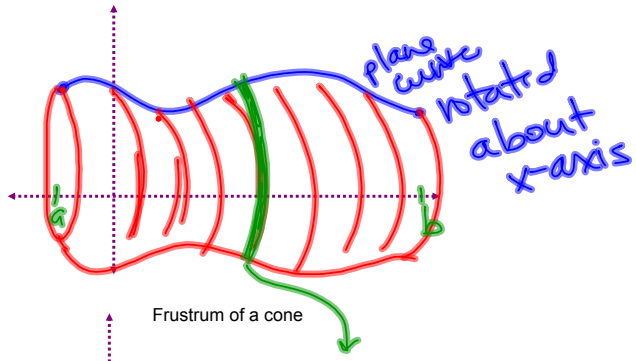
accumulate arc length

$$s(x) = \int_a^x \sqrt{1 + (f'(t))^2}\, dt$$

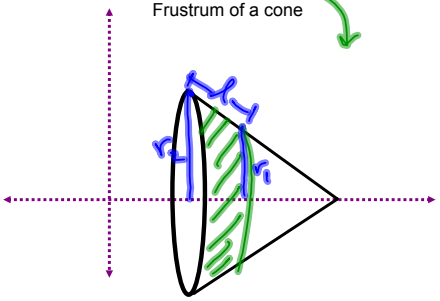
31 Length Curve

Surface Area of a Surface of Revolution

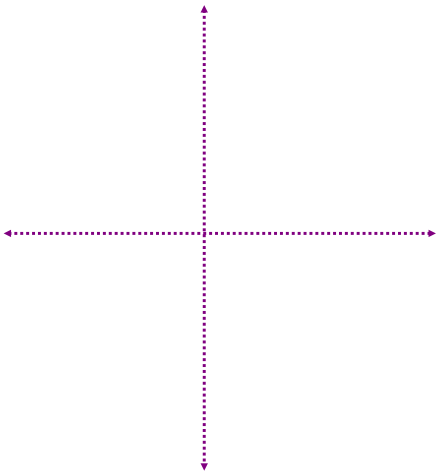
Rotate a plane curve about an axis to create a hollow three-dimensional solid.
Find the surface area of the solid.



Frustrum of a cone



EX 4 Find the area of the surface generated by revolving $y = \sqrt{25-x^2}$ on the interval $[-2,3]$ about the x-axis.



31 Length Curve

EX 5 Find the area of the surface generated by revolving $x = 1-t^2$, $y = 2t$, on the t-interval $[0,1]$ about the x-axis.