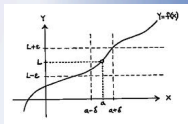
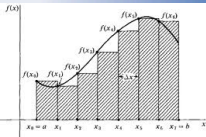


### 3 Limit Theorems



$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

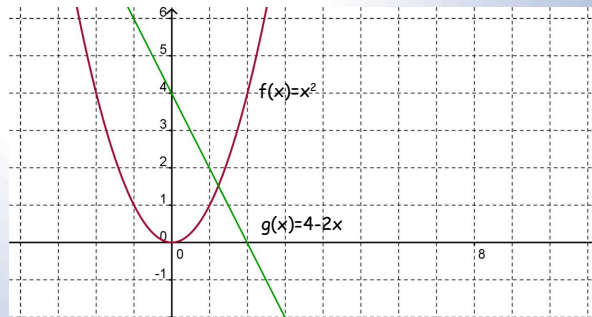
$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$



$$\lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n f(x_i) \Delta x_i = \int_a^b f(x) dx$$

$$\int_a^b f(x) dx = F(b) - F(a)$$

## Calculus: 3 ~ Limit Theorems



$$\lim_{x \rightarrow 1} f(x) - g(x) = ?$$

## Limit Theorems

$n$  is a positive integer.

$k$  is a real number

$f(x)$  &  $g(x)$  have limits as  $x \rightarrow c$

- 1)  $\lim_{x \rightarrow c} k = k$
- 2)  $\lim_{x \rightarrow c} x = c$
- 3)  $\lim_{x \rightarrow c} [kf(x)] = k \lim_{x \rightarrow c} f(x)$
- 4)  $\lim_{x \rightarrow c} [f(x) \pm g(x)] = \lim_{x \rightarrow c} f(x) \pm \lim_{x \rightarrow c} g(x)$
- 5)  $\lim_{x \rightarrow c} [f(x)g(x)] = \lim_{x \rightarrow c} f(x) \lim_{x \rightarrow c} g(x)$
- 6)  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)}$ ,  $g(x) \neq 0$
- 7)  $\lim_{x \rightarrow c} [f(x)]^n = [\lim_{x \rightarrow c} f(x)]^n$
- 8)  $\lim_{x \rightarrow c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow c} f(x)}$ , if  $\lim_{x \rightarrow c} f(x) > 0$  when  $n$  is even.

### 3 Limit Theorems

$$\text{EX 1} \quad \lim_{x \rightarrow 2} (4x^2 - 2x + 1)$$

$$\text{EX 2} \quad \lim_{x \rightarrow -3} \frac{\sqrt{x^2 - 1}}{2x}$$

$$\begin{aligned} \text{EX 3} \quad & \text{If} \quad \lim_{x \rightarrow a} f(x) = 3 \quad \text{and} \quad \lim_{x \rightarrow a} g(x) = -1, \\ & \text{find} \quad \lim_{x \rightarrow a} \frac{2f(x) - 3g(x)}{f(x) + g(x)} \end{aligned}$$

#### **Substitution Theorem**

If  $f(x)$  is a polynomial or a rational function, then  $\lim_{x \rightarrow c} f(x) = f(c)$  assuming  $f(c)$  is defined.

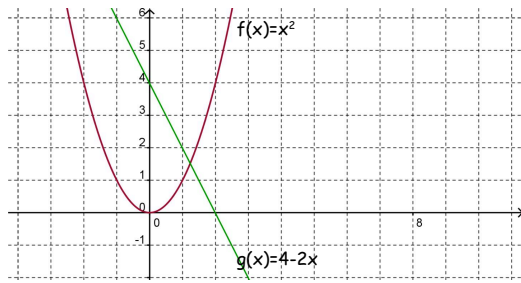
$$\text{Ex 4} \quad \lim_{x \rightarrow -1} \frac{3x^2 - 4x^3 + 7x - 5}{2x^2 + 3x + 4}$$

$$\text{Ex 5} \quad \lim_{x \rightarrow 2} \frac{3x^3 + 4x + 1}{x^2 - x - 2}$$

$$\text{EX 6} \quad \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$$

*Hint: rationalize the numerator.*

### 3 Limit Theorems



$$\lim_{x \rightarrow 1} f(x) =$$

$$\lim_{x \rightarrow 1} g(x) =$$

$$\lim_{x \rightarrow 1} f(x) - g(x) = ?$$

