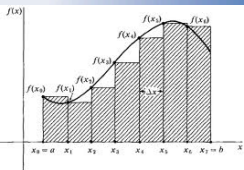


$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

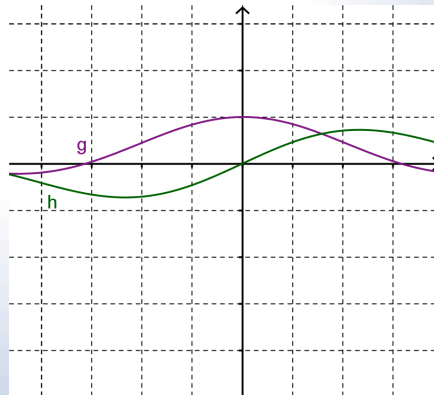
$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$



$$\lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n f(x_i) \Delta x_i = \int_a^b f(x) dx$$

$$\int_a^b f(x) dx = F(b) - F(a)$$

Limits Involving Trigonometric Functions



$$g(t) = \frac{\sin t}{t}$$

$$h(t) = \frac{1 - \cos t}{t}$$

5B Limits Trig Fns

Theorem

For every c in the in the trigonometric function's domain,

$$\lim_{x \rightarrow c} \sin x = \sin c$$

$$\lim_{x \rightarrow c} \csc x = \csc c$$

$$\lim_{x \rightarrow c} \cos x = \cos c$$

$$\lim_{x \rightarrow c} \sec x = \sec c$$

$$\lim_{x \rightarrow c} \tan x = \tan c$$

$$\lim_{x \rightarrow c} \cot x = \cot c$$

Special Trigonometric Limit Theorems

(memorize)

$$\textcircled{1} \quad \lim_{t \rightarrow 0} \frac{\sin t}{t} = 1 = \lim_{t \rightarrow 0} \frac{t}{\sin t}$$

($\frac{0}{0}$ case)

$$\textcircled{2} \quad \lim_{t \rightarrow 0} \frac{1 - \cos t}{t} = 0$$

($\frac{0}{0}$ case)

EX 1 $\lim_{x \rightarrow 0} \frac{3x \tan x}{\sin x}$ = $\lim_{x \rightarrow 0} \left(\frac{3x \cancel{\sin x}}{\cos x \cancel{\sin x}} \right)$
 ($\frac{0}{0}$ case)
 = $\lim_{x \rightarrow 0} \frac{3x}{\cos x} = \frac{0}{1} = 0$

EX 2 $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x}$ = $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \left(\frac{\sin x}{1} \right)$
 ($\frac{0}{0}$ case)
 = $\left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \left(\lim_{x \rightarrow 0} \frac{\sin x}{1} \right)$
 = $1 \cdot 0 = 0$

Note:
 $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

5B Limits Trig Fns

EX 3 $\lim_{\theta \rightarrow 0} \frac{\tan(5\theta)}{\sin(2\theta)} = \lim_{\theta \rightarrow 0} \frac{\sin(5\theta)}{\cos(5\theta) \sin(2\theta)}$

($\frac{0}{0}$ case)

Note:

$$\lim_{\heartsuit \rightarrow 0} \frac{\sin \heartsuit}{\heartsuit} = 1$$

$$= \lim_{\heartsuit \rightarrow 0} \frac{\heartsuit}{\sin \heartsuit}$$

$$= \lim_{\theta \rightarrow 0} \left(\frac{\sin(5\theta)}{5\theta} \right) \left(\frac{2\theta}{\sin(2\theta)} \right) \left(\frac{5}{2\cos(5\theta)} \right)$$

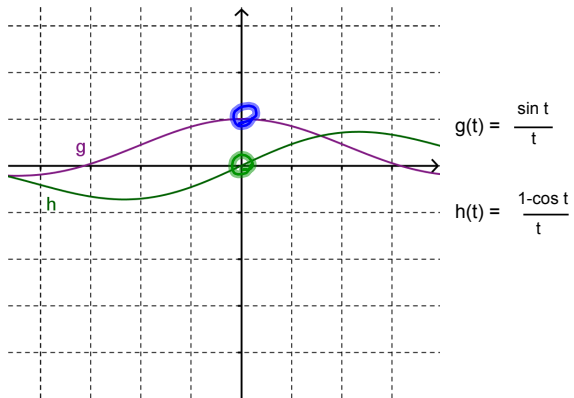
$$= \left(\lim_{5\theta \rightarrow 0} \frac{\sin(5\theta)}{5\theta} \right) \left(\lim_{2\theta \rightarrow 0} \frac{2\theta}{\sin(2\theta)} \right) \left(\lim_{\theta \rightarrow 0} \frac{5}{2\cos(5\theta)} \right)$$

$\heartsuit = 5\theta$ $\heartsuit = 2\theta$ $\frac{5}{2}$

$$= 1 \cdot 1 \cdot \frac{5}{2}$$

$$= \frac{5}{2}$$

5B Limits Trig Fns



Note:

$$\lim_{t \rightarrow 0} \frac{\sin t}{t} = 1$$
$$\lim_{t \rightarrow 0} \frac{1 - \cos t}{t} = 0$$