

# Math 1210 #6

## Continuity

### Definition: Continuity at a Point

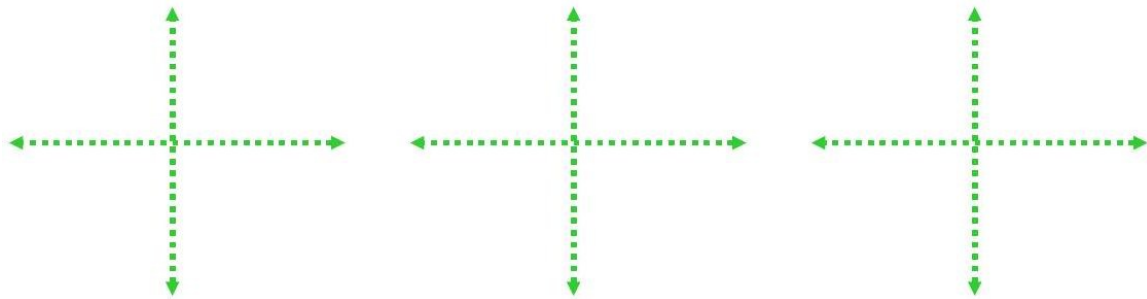
Let  $f$  be defined on an open interval containing  $c$ . We say that  $f$  is continuous at  $c$  if

$$\lim_{x \rightarrow c} f(x) = f(c)$$

This indicates three things:

1. The function is defined at  $x = c$ .
2. The limit exists at  $x = c$ .
3. The limit at  $x = c$  needs to be exactly the value of the function at  $x = c$ .

Three examples:



# Continuous Functions

- a) All polynomial functions are continuous everywhere.
- b) All rational functions are continuous over their domain.
- c) The absolute value function is continuous everywhere.
- d)  $f(x) = \sqrt[n]{x}$  is continuous for all real numbers if  $n$  is odd.
- e)  $f(x) = \sqrt[n]{x}$  is continuous for all non-negative real numbers if  $n$  is even.
- f) The sine and cosine functions are continuous over all real numbers.
- g) The cotangent, cosecant, secant and tangent functions are continuous over their domain.

## More continuous functions

If  $f(x)$  and  $g(x)$  are continuous at  $x = c$ , then so are

$$kf(x), (f \pm g)(x), (fg)(x), \frac{f}{g}(x), (g(x) \neq 0)$$
$$f^n(x), \sqrt[n]{f(x)}, (f(c) > 0 \text{ if } n \text{ is even})$$

### EX 1

State where these functions are continuous.

#### 1a)

$$f(x) = x^2 - 9$$

#### 1b)

$$g(x) = \sqrt{x-5}$$

#### 1c)

$$h(x) = \frac{21-7x}{x-3}$$

#### 1d)

$$p(x) = \begin{cases} 7 - 3x & x \leq 3 \\ -2 & x > 3 \end{cases}$$

# Composite Limit Theorem

If  $\lim_{x \rightarrow c} g(x) = L$  and  $f$  is continuous at  $L$ , then  $\lim_{x \rightarrow c} f(g(x)) = f\left(\lim_{x \rightarrow c} g(x)\right) = f(L)$

## EX 2

At what points are the following functions continuous?

### 2a)

$$h(x) = \frac{1}{\sqrt{4+x^2}}$$

### 2b)

$$g(t) = |t - 2|$$

## EX 3

If  $f(x) = \frac{x^2-49}{x-7}$ , how do we need to complete the definition for this to be continuous everywhere?

# Intermediate Value Theorem

$f$  is a function defined on  $[a, b]$  and  $\omega$  is a number between  $f(a)$  and  $f(b)$ .

If  $f$  is continuous on  $[a, b]$ , then there exists at least one number,  $c$ , ( $a < c < b$ ) such that  $f(c) = \omega$ .

Use interval notation to state all values for which this function is continuous.

