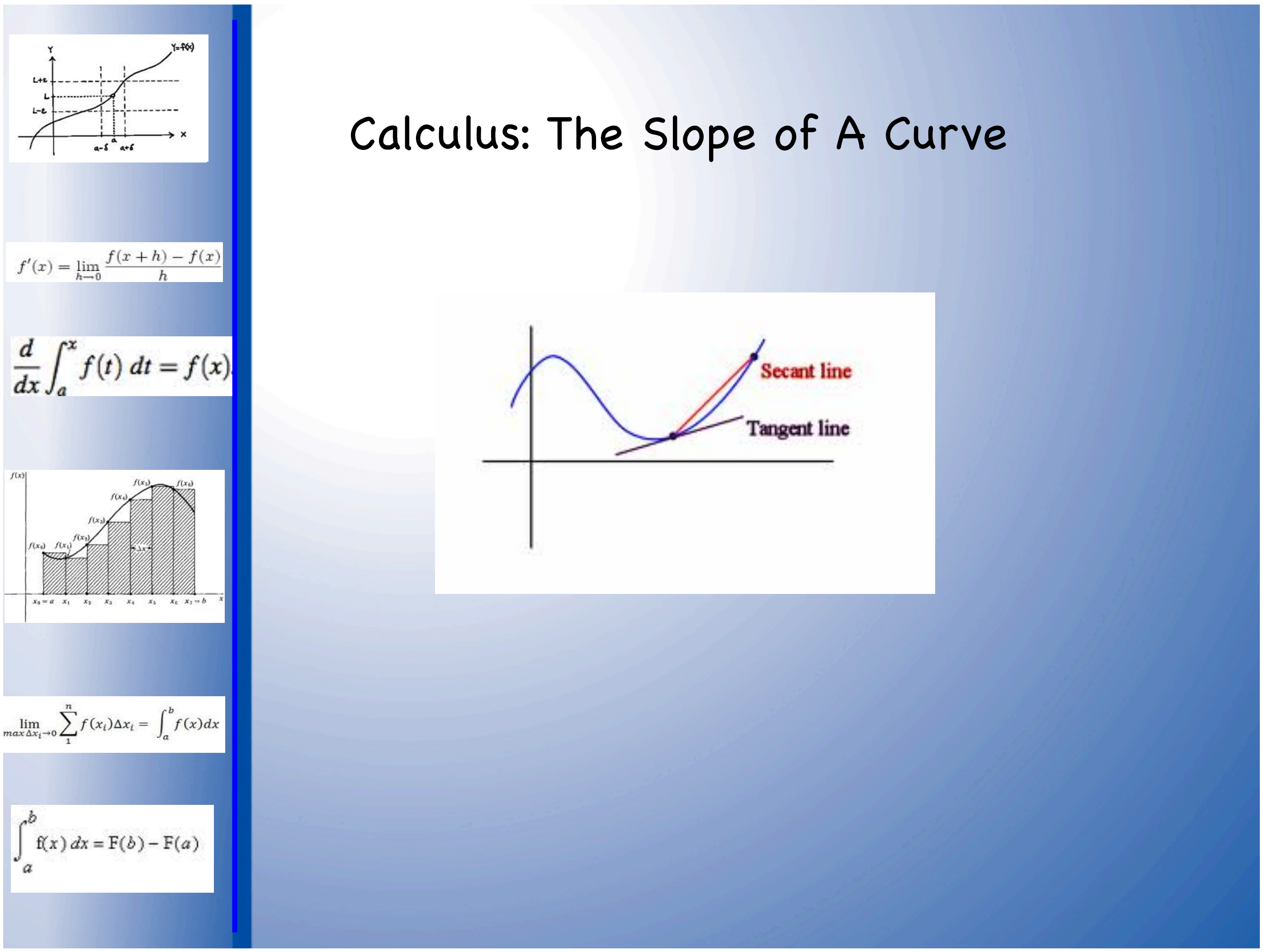
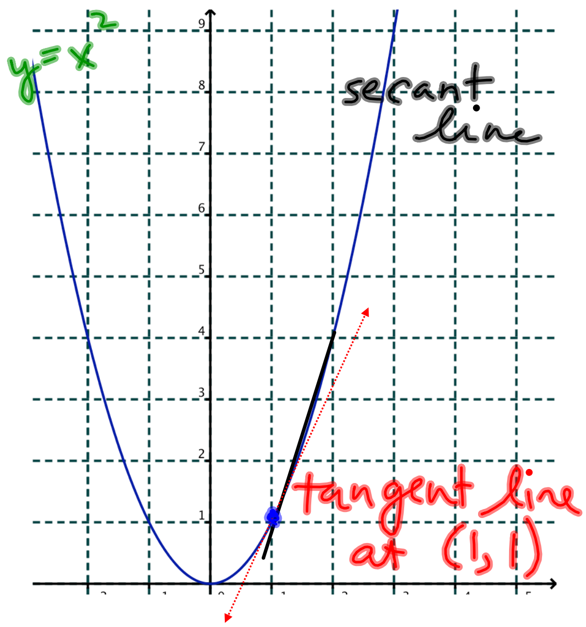

Calculus: The Slope of A Curve
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$
$\lim_{max \Delta x_i \to 0} \sum_1^n f(x_i) \Delta x_i = \int_a^b f(x) dx$
$\int_a^b f(x)\, dx = F(b) - F(a)$
Secant line
Tangent line

How do we find the slope of a curve?



Try to find the slope of this curve at the point (1,1).

fixed
First point (1,1)

Second point:		Slope at that point: (slope of line thru (1,1) and this pt)
(3,9)	$(h=2)$	$m=\frac{9-1}{3-1}=4$
(2,4)	$(h=1)$	$m=\frac{4-1}{2-1}=3$
(1.1, 1.21)	$(h=.1)$	$m=\frac{1.21-1}{1.1-1}=\frac{0.21}{0.1}=2.1$
(1.01, 1.0201)		$m=\frac{1.0201-1}{1.01-1}=\frac{0.0201}{0.01}=2.01$
$(1+h, (1+h)^2)$		$m=2+h$

h = horiz. distance from $x=1$

slope between $(1,1)$ and $(1+h, (1+h)^2)$

$$m=\frac{(1+h)^2-1}{\cancel{1}+h-\cancel{1}}=\frac{\cancel{1}+2h+h^2-\cancel{1}}{h}$$

$$=\frac{2h+h^2}{h}=\frac{\cancel{h}(2+h)}{\cancel{h}}$$

$$=\boxed{2+h}$$

as h gets very small, slope becomes 2

Slope to the left of the origin?

always negative (but changing)

Slope to the right of the origin?

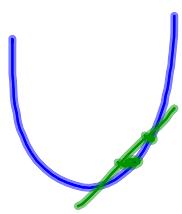
always positive (changing)

EX 1

Find the slope of the curve $y = x^2 - 5x$ at *(2,-6)*

$y = f(x)$

hint: Calculate the slope between (2,-6) and (*2+h, f(2+h)*)



$$m = \frac{f(2+h) - (-6)}{\cancel{2} + h - \cancel{2}}$$

$$= \frac{(2+h)^2 - 5(2+h) + 6}{h}$$

$$= \frac{\cancel{4} + 4h + h^2 - \cancel{10} - 5h + \cancel{6}}{h}$$

$$= \frac{h^2 - h}{h} = \frac{\cancel{h}(h-1)}{\cancel{h}} = \boxed{h-1}$$

as h gets smaller & smaller (approaching 0), $m = -1$

Definition: The slope of a function, *f,* at a point *x* = (*x, f(x)*) is given by

$$\boxed{m = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

"f prime of x"

slope at $(x, f(x))$

$m = \dfrac{f(x+h) - f(x)}{\cancel{x} + h - \cancel{x}}$

$= \dfrac{f(x+h) - f(x)}{h}$

f '(x) is called the <u>derivative</u> of *f* with respect to *x*.

Other names for f '(x):
- slope
- instantaneous rate of change
- speed
- velocity

EX 2
Find the derivative of *f(x) = 4x - 1*

(note: this is a line; w/ slope 4)

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h \to 0} \dfrac{4(x+h) - 1 - (4x - 1)}{h}$

(If I plug in h=0, we get $\frac{0}{0}$ case)

$= \lim\limits_{h \to 0} \dfrac{\cancel{4x} + 4h - \cancel{1} - \cancel{4x} + \cancel{1}}{h}$

$= \lim\limits_{h \to 0} \dfrac{4\cancel{h}}{\cancel{h}} = \lim\limits_{h \to 0} 4 = \boxed{4}$

EX 3

Find the derivative of $f(x) = x^2+4x - 1$

(note: derivative is a formula or fn giving slope at any pt on the curve)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

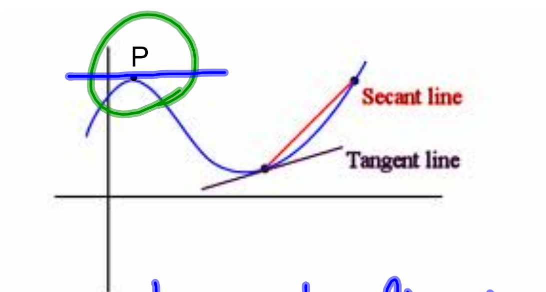
$$= \lim_{h\to 0} \frac{(x+h)^2+4(x+h)-1-(x^2+4x-1)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2+\cancel{4x}+4h-\cancel{1}-\cancel{x^2}-\cancel{4x}+\cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h+4)}{\cancel{h}} = \lim_{h\to 0} 2x+h+4 = \boxed{2x+4}$$

$$\boxed{f'(x) = 2x+4}$$

What is the slope at point *P*?



tangent line is horizontal
w/ $\boxed{m=0}$