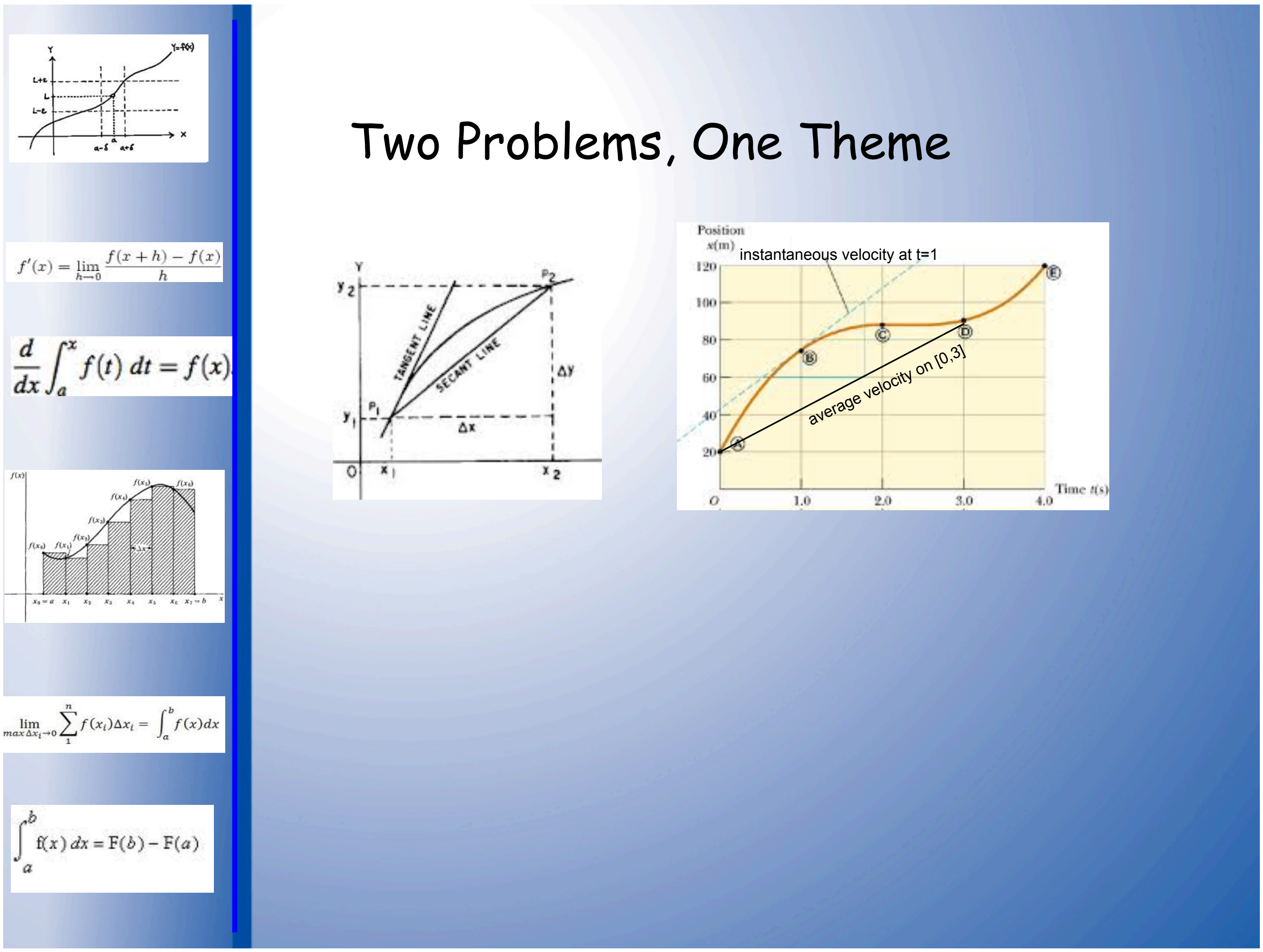
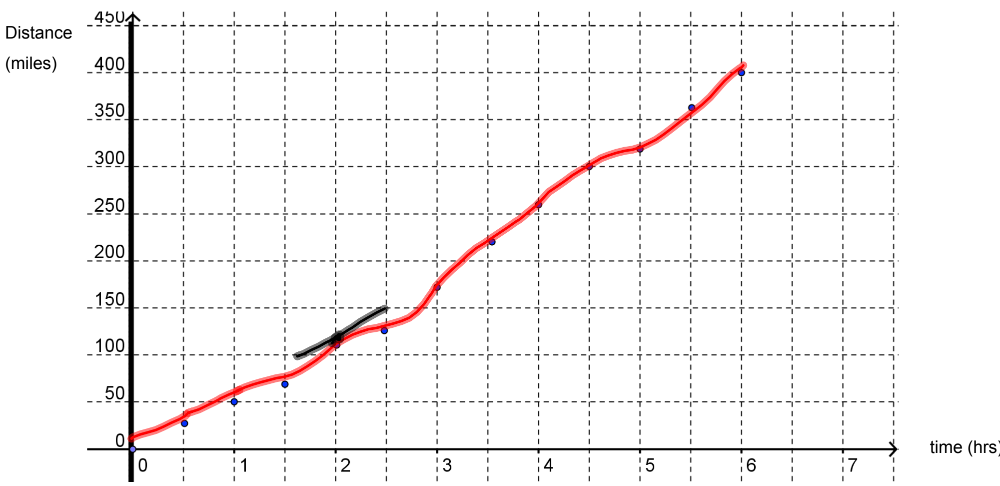

Two Problems, One Theme
TANGENT LINE
SECANT LINE
Δx
Δy
instantaneous velocity at t=1
average velocity on [0,3]
Position x(m)
Time t(s)

8B Two Problems One Theme

It took me 6 hours to drive 400 miles. As I drove I wrote the mileage on the trip-o-meter each half hour. Here is a graph of my trip.



t	d
3	170
2.5	130
2.1	112
2	110

(slope)

What was my average velocity for the trip? $v_{av}= \dfrac{\text{total dist}}{\text{total time}} = \dfrac{400 \text{ mi}}{6 \text{ hr}} = 66.\bar{6} \text{ mi/hr}$

What was my average velocity for the first half of the trip? $V_{half} = \dfrac{170 \text{ mi}}{3 \text{ hr}} = 56.\bar{6} \text{ mi/hr}$

How fast was I going at $t=2$? $v_{inst}=$?

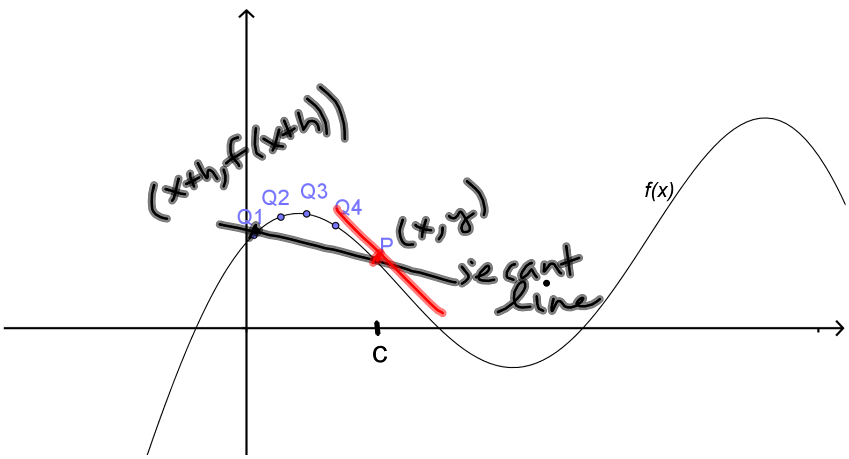
(inst = instantaneous)

close, approx

$$V_{inst} = \frac{112-110}{2.1-2} \frac{\text{mi}}{\text{hr}} = \frac{2}{0.1} = 20 \text{ mi/hr}$$

Archimedes - slope of a tangent line.

Kepler, Galileo, Newton - Instantaneous velocity.



Q = "movable" point.

P = Point in question

secant line⇒ line through P and Q.

tangent line⇒ limiting position (if it exists) of secant line as Q moves closer to P along the curve.

slope of secant line $m_{sec} = \dfrac{f(x+h)-f(x)}{x+h-x} = \dfrac{f(x+h)-f(x)}{h}$

slope of tangent line $m_{tan} = \lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$

8B Two Problems One Theme

EX 1 Find the slope of $y = -x^2 + 3x$ when $x = -1, 2,$ and 5. (find slope formula; the derivative)

$$m_{tan} = f'(x) = \lim_{h\to 0} \frac{-(x+h)^2 + 3(x+h) - (-x^2+3x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{-x^2} - 2xh - h^2 + \cancel{3x} + 3h + \cancel{x^2} - \cancel{3x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-2x-h+3)}{\cancel{h}} = \lim_{h\to 0}(-2x-h+3) = -2x+3$$

at $x=-1$, $m = f'(-1) = -2(-1)+3 = 5$

at $x=2$, $m = f'(2) = -2(2)+3 = -1$

at $x=5$, $f'(5) = -2(5)+3 = -7$

EX 2 Find the equation of the tangent line to $y = \frac{2}{x}$ at $x=1$.

we need a pt and a slope, to find eqn of line

pt: $(1, 2)$ $y = \frac{2}{1} = 2$

$$f'(x) = \lim_{h\to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{2}{(x+h)}\left(\frac{x}{x}\right) - \frac{2}{x}\left(\frac{x+h}{x+h}\right)\right)$$

$LCD = x(x+h)$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{2x - 2(x+h)}{x(x+h)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{\cancel{2x} - \cancel{2x} - 2h}{x(x+h)}\right)$$

$$= \lim_{h\to 0} \frac{-2\cancel{h}}{\cancel{h}x(x+h)} = \lim_{h\to 0} \frac{-2}{x(x+h)}$$

$$= \frac{-2}{x(x+0)}$$

$$f'(x) = \frac{-2}{x^2}$$

slope at $x=1$

$m = f'(1) = \frac{-2}{1^2} = -2$

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pt $(1,2)$, $m=-2$

$y - 2 = -2(x-1)$

$y - 2 = -2x + 2$

$\boxed{y = -2x + 4}$ eqn of tangent line to curve at $x=1$

Geometrically finding the slope of a tangent line to a curve is exactly the same as finding the instantaneous velocity for a moving object.

EX 3 An object travels along a line so that its position is given by $s(t)=t^2+1$ (measured in meters, t measured in seconds.)

a) What is its average velocity on the interval $2 \le t \le 3$?

$$V_{av} = \frac{dist}{time} = \frac{s(3)-s(2)}{3-2} = \frac{(3^2+1)-(2^2+1)}{1} = 5\ m/s$$

b) Average velocity on $2 \le t \le 2.003$?

$$V_{av} = \frac{s(2.003)-s(2)}{2.003-2} = \frac{(2.003^2+1)-(2^2+1)}{0.003} \approx \frac{5.012-5}{0.003} \approx \frac{0.012}{0.003} \approx 4\ m/s$$

c) Average velocity on $2 \le t \le 2+h$?

$$V = \frac{s(2+h)-s(2)}{2+h-2} = \frac{(2+h)^2+1-(2^2+1)}{h} = \frac{\cancel{4}+4h+h^2\cancel{+1}\cancel{-4}\cancel{-1}}{h} = \frac{\cancel{h}(4+h)}{\cancel{h}} = (4+h)\ m/s$$

d) Instantaneous velocity at $t=2$?

$$V_{inst} = \lim_{h\to 0}(4+h) = 4\ m/s$$

"Rate of change" means instantaneous rate of change.

8B Two Problems One Theme