

# Math 1220 #10

## Trigonometric Integrals

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Combining u-substitution and the trigonometric identities, we will address three forms of these integrals.

1.  $\int \sin^n x dx, \int \cos^n x dx$
2.  $\int \sin^m x \cos^n x dx$
3.  $\int \sin(mx)\cos(nx)dx, \int \sin(mx)\sin(nx)dx, \int \cos(mx)\cos(nx)dx$

#### EX 1

$$\int \sin^3 x dx$$

Type 1  
If  $n$  is odd, use  $\sin^2 x + \cos^2 x = 1$ .  
If  $n$  is even, use half - angle formulas.

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

**EX 2**

$$\int \cos^4 x dx$$

Type 1

If  $n$  is odd, use  $\sin^2 x + \cos^2 x = 1$ .  
If  $n$  is even, use half - angle formulas.

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

**EX 3**

$$\int \cos^5 x \sin^{-4} x dx$$

Type 2

If  $m$  or  $n$  is odd and positive, factor out  $\sin x$  or  $\cos x$   
and use  $\sin^2 x + \cos^2 x = 1$ .  
If  $m$  and  $n$  are even and positive, use half - angle identities.

**EX 4**

$$\int \cos^2 x \sin^4 x dx$$

Type 2

If  $m$  or  $n$  is odd and positive, factor out  $\sin x$  or  $\cos x$   
and use  $\sin^2 x + \cos^2 x = 1$ .

If  $m$  and  $n$  are even and positive, use half - angle identities.

**EX 5**

$$\int \sin(4x)\cos(5x)dx$$

Type 3

Use product identities :

$$\sin(mx)\cos(nx) = \frac{1}{2}[\sin((m+n)x) + \sin((m-n)x)]$$

$$\sin(mx)\sin(nx) = -\frac{1}{2}[\cos((m+n)x) - \cos((m-n)x)]$$

$$\cos(mx)\cos(nx) = \frac{1}{2}[\cos((m+n)x) + \cos((m-n)x)]$$

**EX 6**

$$\int_{-4}^4 \sin\left(\frac{m\pi x}{4}\right) \sin\left(\frac{n\pi x}{4}\right) dx$$