

Math 1220 #11

Rationalizing and Trigonometric Substitutions

Rationalizing Substitutions

Integrands involving $\sqrt[n]{ax + b}$

EX 1

$$\frac{x^2 + 3x}{\sqrt{x + 4}} dx$$

EX 2

$$\int \frac{\sqrt{x}}{x + 1} dx$$

Integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, $\sqrt{x^2 - a^2}$, $a \in \mathfrak{R}$

a)

$$\sqrt{a^2 - x^2} \rightarrow \text{let } x = a \sin \theta \quad \theta \in [-\pi/2, \pi/2]$$

b)

$$\sqrt{a^2 + x^2} \rightarrow \text{let } x = a \tan \theta \quad \theta \in (-\pi/2, \pi/2)$$

c)

$$\sqrt{x^2 - a^2} \rightarrow \text{let } x = a \sec \theta \quad \theta \in [0, \pi], \theta \neq \pi/2$$

EX 3

$$\int \frac{x^2}{\sqrt{16 - x^2}} dx$$

EX 4

$$\int_2^3 \frac{dt}{t^2 \sqrt{t^2 - 1}}$$

Completing the Square

(Use this strategy when there is a quadratic expression in the radical.)

EX 5

$$\int \frac{3x}{\sqrt{x^2 + 4x - 5}} dx$$