Math 1220 #16
Improper Integrals

Improper Integral

It is like a definite integral except one or both of the limits of integration are $\pm^{\infty}$.

Definition

$$\int_{-\infty}^{b} f(x)dx = \lim_{a\to-\infty} \int_{a}^{b} f(x)dx$$

$$\int_{a}^{\infty} f(x)dx = \lim_{b\to\infty} \int_{a}^{b} f(x)dx$$

converge if the limit exists and is finite.

diverge if the limit does not exist (or goes to $\pm\infty$).

EX 1

$$\int_{-\infty}^{2} e^{x}dx$$

EX 2

$\int_1^\infty \frac{1}{\sqrt{\pi x}} dx$

EX 3

$\int_1^\infty \frac{x}{(1+x^2)^2} dx$

Definition

If $\int_{-\infty}^{0} f(x)dx$ and $\int_0^\infty f(x)dx$ converge,
then $\int_{-\infty}^{\infty} f(x)dx$ converges and

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_0^\infty f(x)dx$$

otherwise, $\int_{-\infty}^{\infty} f(x)dx$ diverges.

EX 4

$$\int_1^\infty \frac{1}{x^p}\,dx$$

EX 5

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 16)}$$

EX 6

$$\int_{-\infty}^{\infty} \frac{x}{\sqrt{x^2 + 16}}\, dx$$