Math 1220 #19
Infinite Series

Zeno's Paradox says that if you step from 0 to 1/2, then keep taking steps halfway between where you are and 1 that you will never get to 1.

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}$$

Let S_i be the partial sum of the first i terms in the sequence.

$$S_1=\frac{1}{2}=$$
$$S_2=\frac{1}{2}+\frac{1}{4}=$$
$$S_3=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=$$
$$\vdots$$
$$S_n=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}=$$
$$\lim_{n\to\infty}\left(1-\frac{1}{2^n}\right)=$$

Infinite Series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots = \sum_{i=1}^{\infty} a_i$
Partial Sum $\sum_{i=1}^{n} a_i = S_n$

Definition

$\sum a_i$ converges and has a sum, S, if the sequence of partial sums converges to S, i.e. $\lim_{n\to\infty} S_n = S$.
If $\{S_n\}$ diverges, then the series diverges and has no sum.

Geometric Series

$a \neq 0 \sum_{i=1}^{\infty} ar^{i-1} = a + ar + ar^2 + ar^3 + \cdots$

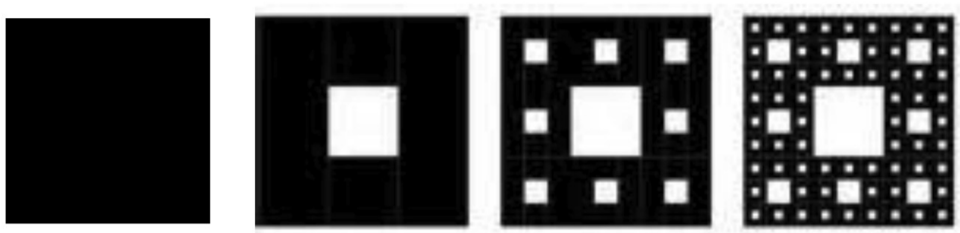
EX 1

Show that a geometric series converges for at least some r and find its sum.

$$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^n$$

EX 2

If this pattern continues indefinitely, what fraction of the original square will eventually be unshaded?



Theorem

n^th term test for divergence
If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.
If $\lim_{n\to\infty} a_n \neq 0$ or if $\lim_{n\to\infty} a_n$ DNE, then the series diverges.

EX 3

Does $\sum_{i=1}^{\infty} \frac{3i-7}{4i+3}$ converge or diverge?

Harmonic Series $\sum_{n=1}^{\infty} \frac{1}{n} =$

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$$
$$\lim_{n \to \infty} \frac{1}{n} = 0$$

but does it converge?

EX 4

does $\sum_{i=1}^{\infty} \frac{3}{i(i+1)}$ converge or diverge?

Linearity of a Convergent Positive Series

If $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ both converge,

then $\sum_{i=1}^{\infty} ca_i = c\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} (a_i + b_i) = \sum_{i=1}^{\infty} a_i + \sum_{i=1}^{\infty} b_i$

also converge.

EX 5

Does $\sum_{k=1}^{\infty} \left[5\left(\frac{1}{2}\right)^k - 3\left(\frac{1}{7}\right)^k\right]$ diverge or converge?

Theorem

If $\sum_{k=1}^{\infty} a_k$ diverges and $c \neq 0$, then $\sum_{k=1}^{\infty} ca_k$ diverges.

Grouping Terms in an infinite series

The terms in a convergent positive series can be grouped in any way and the new series will still converge to the same sum.

Why don't we just use computers to tell if a series converges?

Consider the Harmonic Series:

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

On a computer, for $\mathrm{n} = 10^{43}$, $\mathrm{S_n} = 100$ and $\mathrm{S}_{272,000,000} \cong 20$.