

$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$   
 or  
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$

Then  
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

provided that the latter limit exists.

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$   
 $g(x) = g(a) + g'(a)(x-a) + \frac{g''(a)}{2!}(x-a)^2 + \dots$

$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$

$\int u dv = uv - \int v du$

where  $u$  and  $v$  are functions of  $x$ .

**The Natural Logarithmic Function**

### The Natural Logarithmic Function

$$D_x \left( \frac{x^3}{3} \right) = x^2$$

$$D_x \left( \frac{x^2}{2} \right) = x$$

$$D_x (x) = 1$$

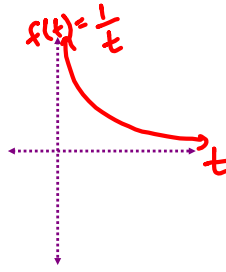
$$D_x (?) = \frac{1}{x}$$

$$D_x \left( \frac{-1}{x} \right) = \frac{1}{x^2}$$

$$D_x \left( \frac{-1}{2x^2} \right) = \frac{1}{x^3}$$

Definition

$$\ln x = \int_1^x \frac{1}{t} dt, \quad x > 0$$



From the First Fundamental Theorem of Calculus

$$D_x \left( \int_1^x \frac{1}{t} dt \right) = D_x(\ln x) = \frac{1}{x}, \quad x > 0$$

EX 1 Find  $\frac{dy}{dx}$  if  $y = \ln(x^2)$

EX 2 Find  $\frac{dy}{dx}$  and state the domain

a)  $y = \ln(\sqrt[3]{2x})$

b)  $y = \ln(3x^2 + 14x - 5)$

$$D_x[\ln|x|] = \frac{1}{x} \quad x \neq 0$$

Proof

EX 3 Evaluate the integrals.

a)  $\int \frac{6}{3x-2} dx$

b)  $\int_2^5 \frac{3x}{7-2x^2} dx$       Note: This integral is valid because  $7-2x^2 \neq 0$  on  $[2,5]$

## Properties of Logarithms

$$\begin{aligned}\ln 1 &= 0 & \ln(ab) &= \ln a + \ln b \\ \ln a^r &= r \ln a & \ln\left(\frac{a}{b}\right) &= \ln a - \ln b\end{aligned}$$

Proof

EX 4 Rewrite as a single logarithmic expression.

$$\ln(x^2 - 9) - 2\ln(x - 3) - \ln(x + 3)$$

EX 5 Find  $\frac{dy}{dx}$  by logarithmic differentiation  $y = \frac{(x^2 + 3)^{\frac{2}{3}} (3x + 2)^2}{\sqrt{x + 1}}$