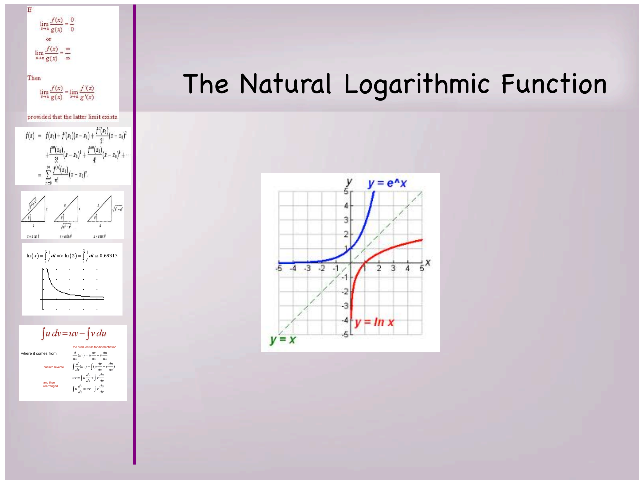


The Natural Logarithmic Function

$$D_x\left(\frac{x^3}{3}\right)=x^2$$

$$D_x\left(\frac{x^2}{2}\right)=x$$

$$D_x\left(x\right)=1$$

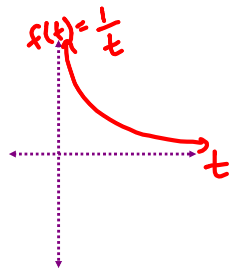
$$D_x\left(?\right)=\frac{1}{x}$$

$$D_x\left(\frac{-1}{x}\right)=\frac{1}{x^2}$$

$$D_x\left(\frac{-1}{2x^2}\right)=\frac{1}{x^3}$$

Definition

$$\ln x = \int_1^x \frac{1}{t}\,dt, \quad x > 0$$



From the First Fundamental Theorem of Calculus

$$D_x\left(\int_1^x \frac{1}{t}\,dt\right) = D_x(\ln x) = \frac{1}{x}, \quad x > 0$$

EX 1 Find $\frac{dy}{dx}$ if $y = ln\,(x^2)$

EX 2 Find $\frac{dy}{dx}$ and state the domain

a) $y = ln\left(\sqrt[3]{2x}\right)$

b) $y = \ln(3x^2 + 14x - 5)$

$$D_x\left[\ln|x|\right]=\frac{1}{x} \quad x \neq 0$$

Proof

EX 3 Evaluate the integrals.

a) $\int \frac{6}{3x-2}\,dx$

b) $\int_2^5 \frac{3x}{7-2x^2}\,dx$ Note: This integral is valid because $7-2x^2 \neq 0$ on $[2,5]$

Properties of Logarithms

$$\ln 1 = 0 \qquad \ln(ab) = \ln a + \ln b$$

$$\ln a^r = r \ln a \qquad \ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

Proof

EX 4 Rewrite as a single logarithmic expression.

$$\ln\left(x^2 - 9\right) - 2\ln\left(x - 3\right) - \ln\left(x + 3\right)$$

EX 5 Find $\frac{dy}{dx}$ by logarithmic differentiation $y = \frac{\left(x^2+3\right)^{\frac{2}{3}}\left(3x+2\right)^2}{\sqrt{x+1}}$