

Math 1220 #1

The Natural Logarithmic Function

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$$D_x \left(\frac{x^3}{3} \right) = x^2$$

$$D_x \left(\frac{x^2}{2} \right) = x$$

$$D_x(x) = 1$$

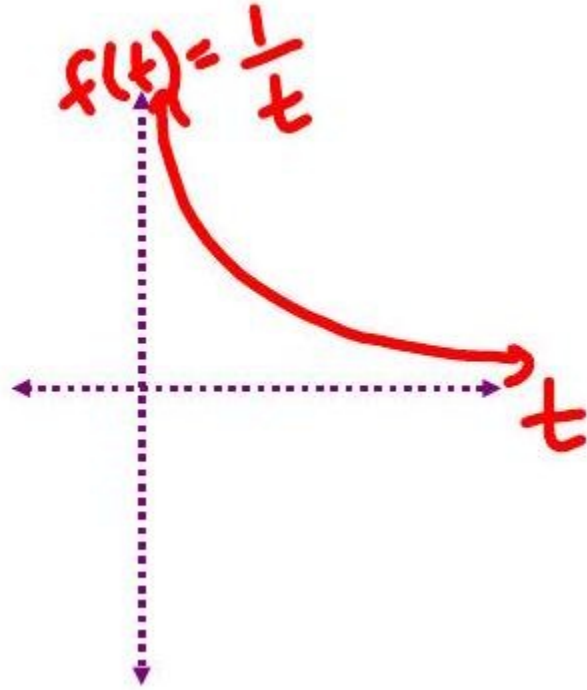
$$D_x(?) = \frac{1}{x}$$

$$D_x \left(\frac{-1}{x} \right) = \frac{1}{x^2}$$

$$D_x \left(\frac{-1}{2x^2} \right) = \frac{1}{x^3}$$

Definition

$$\ln x = \int_1^x \frac{1}{t} dt, x > 0$$



From the First Fundamental Theorem of Calculus

$$D_x \left(\int_1^x \frac{1}{t} dt \right) = D_x(\ln x) = \frac{1}{x}, x > 0$$

EX 1

Find $\frac{dy}{dx}$ if $y = \ln(x^2)$

EX 2

Find $\frac{dy}{dx}$ and state the domain

2a)

$$y = \ln(\sqrt[3]{2x})$$

2b)

$$y = \ln(3x^2 + 14x - 5)$$

$$D_x[\ln |x|] = \frac{1}{x} \quad x \neq 0$$

Proof

EX 3

Evaluate the integrals.

3a)

$$\int \frac{6}{3x-2} dx$$

3b)

$$\int_2^5 \frac{3x}{7-2x^2} dx$$

Note: This integral is valid because $7 - 2x^2 \neq 0$ on $[2,5]$

Properties of Logarithms

$$\begin{aligned}\ln 1 &= 0 & \ln(ab) &= \ln a + \ln b \\ \ln a^r &= r \ln a & \ln\left(\frac{a}{b}\right) &= \ln a - \ln b\end{aligned}$$

Proof

EX 4

Rewrite as a single logarithmic expression.

$$\ln(x^2 - 9) - 2\ln(x - 3) - \ln(x + 3)$$

EX 5

Find $\frac{dy}{dx}$ by logarithmic differentiation $y = \frac{(x^2+3)^{\frac{2}{3}}(3x+2)^2}{\sqrt{x+1}}$