

**Math 1220 #20**  
**Positive Series: Integral Test**

## **Positive Series: Integral Test**

### **Bounded Sum Test**

A series  $\sum a_i$  of nonnegative terms converges if and only if its partial sums are bounded above.

#### **EX 1**

Does  $\sum_{k=1}^{\infty} \frac{|\sin k|}{(k+1)!}$  converge?

# Integral Test

If  $f(x)$  is continuous, positive and nonincreasing on  $[N, \infty)$  and  $a_k = f(k)$  for all positive integers,  $k$ , then  $\sum_{n=N}^{\infty} a_n$  converges if and only if  $\int_N^{\infty} f(x)dx$  converges.

## EX 2

Does  $\sum_{k=1}^{\infty} \frac{5k^2}{1+k^3}$  converge or diverge?

## *p*-series test

$\sum_{k=1}^{\infty} \frac{1}{k^p}$  is called a *p*-series. It converges if  $p > 1$  and diverges if  $p \leq 1$ .

**EX 3**

Does  $\sum_{k=1}^{\infty} \frac{1}{k^3}$  converge or diverge?

**EX 4**

Estimate the error made by approximating the series by the sum of the first five terms.

$$E_n = \sum_{k=n}^{\infty} \frac{1}{k\sqrt{k}} \quad S_n = \sum_{k=1}^n \frac{1}{k\sqrt{k}}$$