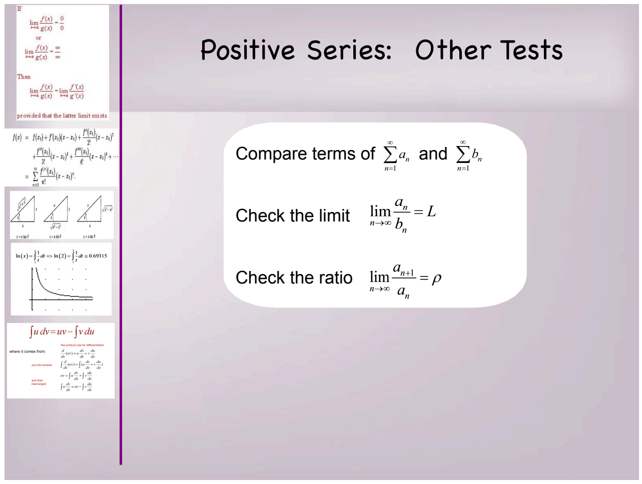

Two types of series may converge.

Geometric Series: $\sum_{n=1}^{\infty} ar^n$ converges if $|r|<1$.

p-series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p>1$.

<u>Ordinary Comparison Test</u>

If $0 \le a_n \le b_n$ for every $n \ge N$

(i) If $\sum_{n=1}^{\infty} b_n$ converges, so does $\sum_{n=1}^{\infty} a_n$.

(ii) If $\sum_{n=1}^{\infty} a_n$ diverges, so does $\sum_{n=1}^{\infty} b_n$.

EX 1 Does $\sum_{n=1}^{\infty} \frac{3n+4}{4n^2-2n-5}$ converge or diverge?

Limit Comparison Test

Assume $a_n \geq 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

If $0 < L < \infty$, then $\sum a_n$ and $\sum b_n$ converge or diverge together.

If $L = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

EX 2 Does this series converge or diverge?

$$\frac{1}{1^2+1} + \frac{2}{2^2+1} + \frac{3}{3^2+1} + \ldots$$

EX 3 Does this series converge or diverge? $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2+1}$

Ratio Test

If $\sum a_n$ is a series of positive terms and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$,

then *i)* if $\rho < 1$, the series converges.

ii) if $\rho > 1$ or if $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \infty$, the series diverges.

iii) if $\rho = 1$, then the test is inconclusive.

EX 4 Does this series converge or diverge? $3 + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!} + ...$

EX 5 Does this series converge or diverge? $\sum_{n=1}^{\infty} \frac{n!}{5+n}$