

# Math 1220 #21

## Positive Series: Other Tests

Two types of series may converge.

Geometric series:  $\sum_{n=1}^{\infty} ar^n$  converges if  $|r| < 1$ .

$p$ -series:  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converges if  $p > 1$ .

### Ordinary Comparison Test

If  $0 \leq a_n \leq b_n$  for every  $n \geq N$

- (i) If  $\sum_{n=1}^{\infty} b_n$  converges, so does  $\sum_{n=1}^{\infty} a_n$ .
- (ii) If  $\sum_{n=1}^{\infty} a_n$  diverges, so does  $\sum_{n=1}^{\infty} b_n$ .

#### EX 1

Does  $\sum_{n=1}^{\infty} \frac{3n+4}{4n^2-2n-5}$  converge or diverge?

### Limit Comparison Test

Assume  $a_n \geq 0$  and  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ ,  $0 < L < \infty$ , then  $\sum_{n \rightarrow \infty} a_n$  and  $\sum b_n$  converge or diverge together. If  $L = 0$  and  $\sum b_n$  converges, then  $\sum a_n$  converges.

#### EX 2

Does this series converge or diverge?

$$\frac{1}{1^2 + 1} + \frac{2}{2^2 + 1} + \frac{3}{3^2 + 1} + \dots$$

### EX 3

Does this series converge or diverge?  $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2+1}$

## Ratio Test

If  $\sum a_n$  is a series of positive terms and  $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \rho$ , then

- i) if  $\rho < 1$ , the series converges.
- ii) if  $\rho > 1$  or if  $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \infty$ , the series diverges.
- iii) if  $\rho = 1$ , then the test is inconclusive.

### EX 4

Does this series converge or diverge?  $3 + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!} + \dots$

**EX 5**

Does this series converge or diverge?  $\sum_{n=1}^{\infty} \frac{n!}{5+n}$