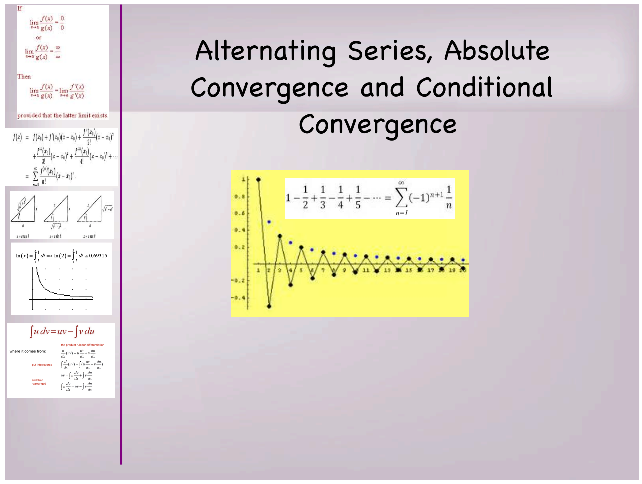

In an Alternating Series, every other term has the opposite sign.

$$a_1 - a_2 + a_3 - a_4 + a_5 - \dots$$

<u>AST (Alternating Series Test)</u>

Let $a_1 - a_2 + a_3 - a_4 + \dots$ be an alternating series such that $a_n > a_{n+1} > 0$, then the series converges.

The error made by estimating the sum, S_n is less than or equal to a_{n+1}, i.e. $E = |S - S_n| \le a_{n+1}$.

EX 1 Does an Alternating Harmonic Series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

EX 2 Does this series diverge or converge? What is the error estimate made when approximating the sum using S_6?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2+1}$$

Absolute Convergence

If $\sum |u_n|$ converges, then $\sum u_n$ converges.

EX 3 Does $2+\frac{2}{2^3}+\frac{2}{3^3}-\frac{2}{4^3}+\frac{2}{5^3}+\frac{2}{6^3}+\frac{2}{7^3}-\frac{2}{8^3}+\ldots$ converge or diverge?

Absolute Ratio Test

Let $\sum a_n$ be a series of nonzero terms and suppose $\lim_{n\to\infty} \frac{|u_{n+1}|}{|u_n|} = \rho$.

i) if $\rho < 1$, the series converges absolutely.

ii) if $\rho > 1$, the series diverges.

iii) if $\rho = 1$, then the test is inconclusive.

EX 4 Show $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{e^n}$ converges absolutely.

Conditional Convergence

$\sum u_n$ is conditionally convergent if $\sum u_n$ converges but $\sum |u_n|$ does not.

EX 5 Classify as absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1} + \sqrt{n}}$$

Rearrangement Theorem

The terms of an absolutely convergent series can be rearranged without affecting either the convergence or the sum of the series.