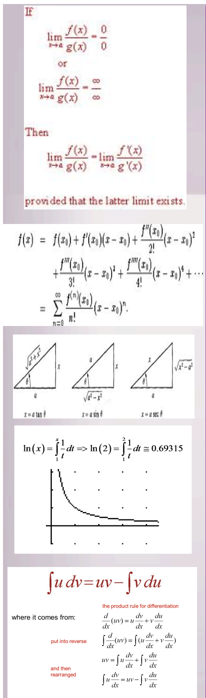

Operations on Power Series

$$\frac{1}{1-x} = 1+x+x^2+x^3+\ldots \quad -1<x<1$$

$$\frac{1}{(1-x)^2} = 1+2x+3x^2+4x^3+\ldots \quad -1<x<1$$

Operations on Power Series

Think of a power series as a polynomial with infinitely many terms.

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$

Theorem A

Let $S(x) = \sum_{n=0}^{\infty} a_n x^n$ on the interval, I.

If x is interior to I, then

1) $S'(x) = \sum_{n=0}^{\infty} D_x\left(a_n x^n\right) = \sum_{n=0}^{\infty} n a_n x^{n-1}$

2) $\int_0^x S(t)\,dt = \sum_{n=0}^{\infty} \int_0^x a_n t^n\,dt = \sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1}$

EX 1 We know

$$1 + x + x^2 + x^3 + \ldots = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad x \in (-1,1)$$

EX 2 Show $S'(x)=S(x)$ for $S(x)=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}...$.

You must first demonstrate convergence, then solve $S'(x)=S(x)$.

Notice $S(0) = 1$.

EX 3 Find the power series for $f(x)=\frac{x}{1+x^2}$.

Theorem B

If $f(x) = \sum a_n x^n$ and $g(x) = \sum b_n x^n$ with both series converging

for $|x| < r$, we can perform arithmetic operations and the resulting series will converge for $|x| < r$. (If $b_0 \neq 0$, the result holds for division, but we can guarantee its validity only for $|x|$ sufficiently small.)

EX 4 Find a power series for $f(x) = sinh(x)$.

EX 5 Find the power series for $f(x) = \dfrac{\arctan(x)}{1 + x^2 + x^4}$.

EX 6 Find these sums.

a) $1 + x^2 + x^4 + x^6 + x^8 + ...$

b) $\cos x + \cos^2 x + \cos^3 x + \cos^4 x + ...$