Math 1220 #24
Operations on Power Series

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Think of a power series as a polynomial with infinitely many terms.

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

Theorem A

Let $S(x) = \sum_{n=0}^{\infty} a_n x^n$ on the interval, I.
If x is interior to I, then

1. $S'(x) = \sum_{n=0}^{\infty} D_x(a_n x^n) = \sum_{n=0}^{\infty} n a_n x^{n-1}$
2. $\int_0^x S(t)dt = \sum_{n=0}^{\infty} \int_0^x a_n t^n dt = \sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1}$

EX 1

We know

$$1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad x \in (-1,1)$$

EX 2

Show $S'(x) = S(x)$ for $S(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \ldots.$

You must first demonstrate convergence, then solve $S'(x) = S(x)$. Notice $S(0) = 1$.

EX 3

Find the power series for $f(x) = \frac{x}{1+x^2}$.

Theorem B

If $f(x) = \sum\ a_n x^n$ and $g(x) = \sum\ b_n x^n$ with both series converging for $|x| < r$, we can perform arithmetic operations and the resulting series will converge for $|x| < r$.

(If $b_0 \neq 0$, the result holds for division, but we can guarantee its validity only for $|x|$ sufficiently small.)

EX 4

Find a power series for $f(x) = \sinh(x)$.

EX 5

Find the power series for $f(x) = \frac{\arctan(x)}{1+x^2+x^4}$.

EX 6

Find these sums.

6a)

$1 + x^2 + x^4 + x^6 + x^8 + \cdots$

6b)

$\cos x + \cos^2 x + \cos^3 x + \cos^4 x + \cdots$