Math 1220 #6
Inverse Trigonometry Functions and Their Derivatives

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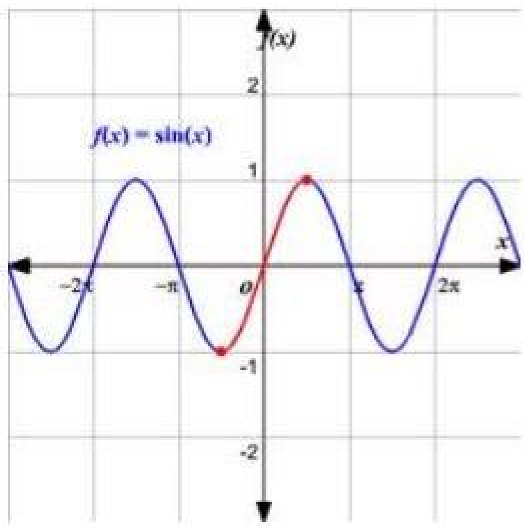
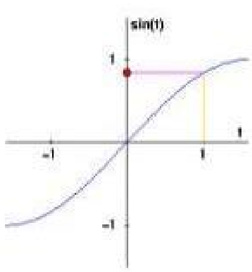
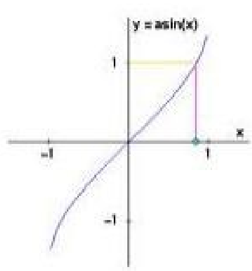
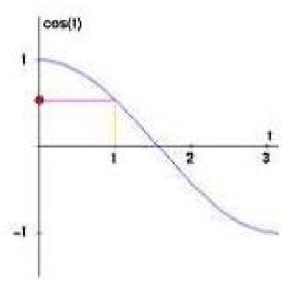
The graph of $y = \sin x$ does not pass the horizontal line test, so it has no inverse.

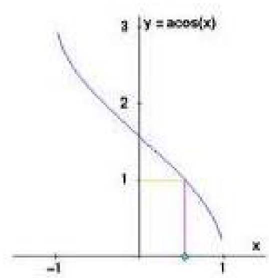


If we restrict the domain (to half a period), then we can talk about an inverse function.









Definition

$$x = \sin^{-1}\ y \Leftrightarrow y = \sin\ x \qquad x\epsilon\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$
$$x = \cos^{-1}\ y \Leftrightarrow y = \cos\ x \qquad x\epsilon[0,\pi]$$

EX 1

Evaluate these without a calculator.

1a)

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

1b)

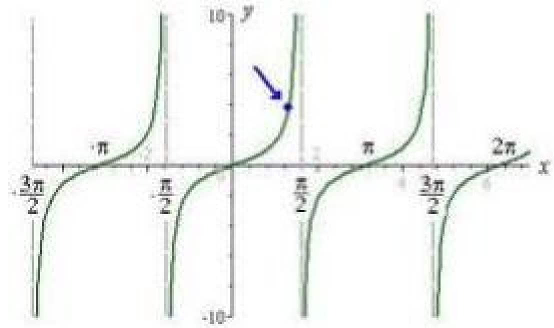
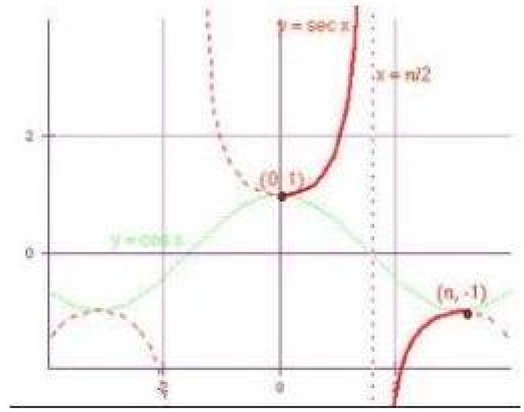
$\sin^{-1}(\sin(3\pi/2))$

1c)

$\sin^{-1}(1)$

1d)

$\cos^{-1}(\cos(-\pi/4))$

<table>
<tr><th>$y = \tan\ x$</th><th>$y = \sec\ x$</th></tr>
<tr><td>
</td><td>
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Definition

$$x = \tan^{-1}\ x \Leftrightarrow y = \tan\ x \quad x \in (-\pi/2, \pi/2)$$
$$x = \sec^{-1}\ y \Leftrightarrow y = \sec\ x \quad x \in [0, \pi/2) \cup (\pi/2, \pi)$$

EX 2

Evaluate without a calculator.

2a)

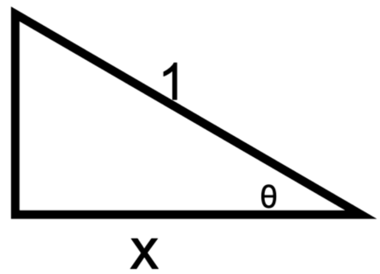
$\tan^{-1}(-1)$

2b)

$\sec^{-1}(2)$

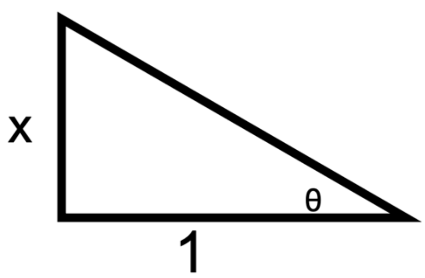
2c)

$$\mathbf{arctan}\left(\tan\left(\frac{3\pi}{4}\right)\right)$$



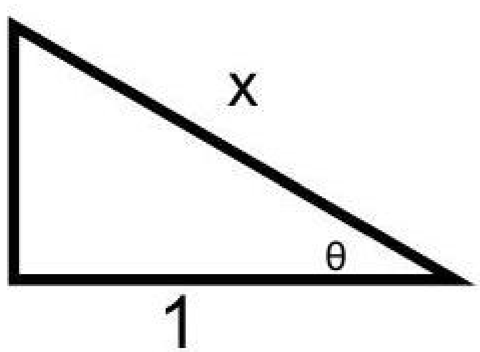
$\theta = \cos^{-1} x$

$\sin(\cos^{-1} x) =$



$\theta = \tan^{-1} x$

$\sec(\tan^{-1} x) =$



$\theta = \sec^{-1} x$

$\tan(\sec^{-1} x) =$

EX 3

Calculate $\sin\left[2\cos^{-1}(1/4)\right]$ with no calculator.

Derivatives of Inverse Trig Functions

$D_x[\sin x] = \cos x$	$D_x[\cos x] = -\sin x$
$D_x[\tan x] = \sec^2 x$	$D_x[\cot x] = -\csc^2 x$
$D_x[\sec x] = \sec x \tan x$	$D_x[\csc x] = -\csc x \cot x$

Let $y = \cos^{-1} x$. Find y'.

EX 4

$D_x\left(tan^{-1}(5x^2 - 3x + 1)\right) =$

$$\begin{pmatrix} D_x(sin^{-1}\ x) = \frac{1}{\sqrt{1-x^2}}, x_{\in}(-1,1) & D_x(cos^{-1}\ x) = \frac{-1}{\sqrt{1=x^2}} \\ D_x(sec^{-1}\ x) = \frac{1}{|x|\ \sqrt{x^2-1}}, |x| > 1 & D_x(tan^{-1}\ x) = \frac{1}{1+x^2} \end{pmatrix}$$

EX 5

Evaluate these integrals.

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$$
$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$
$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1} |x| + C$$

5a)

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx$$

5b)

$$\int \frac{e^x}{1+e^{2x}}\,dx$$