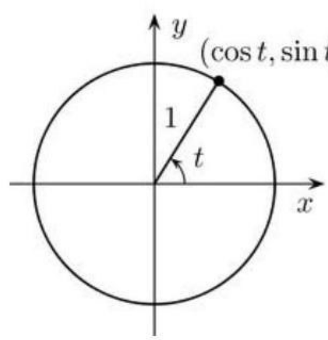
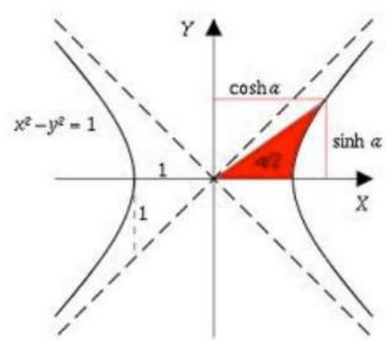
Math 1220 #7
The Hyperbolic Functions

Hyperbolic Functions

$$\sinh\ x = \frac{e^x - e^{-x}}{2} \quad \text{(culled "Sinsh")}$$
$$\cosh\ x = \frac{e^x + e^{-x}}{2}$$
$$\tanh\ x = \frac{\sinh\ x}{\cosh\ x}$$

Relation to the trigonometric functions

1. $(\cos\ \theta, \sin\ \theta)$ is a point on the unit circle. 2. $y = \sin\ x$ is an odd function. 3. $y = \cos\ x$ is an even function. 4. $\sin^2\ \theta + \cos^2\ \theta = 1$  	1. $(\cosh\ \theta, \sinh\ \theta)$ is a point on the unit hyperbola. 2. $y = \sinh\ x$ is an odd function. 3. $y = \cosh\ x$ is an even function. 4. $\cosh^2 \theta - \sinh^2 \theta = 1$ 

Prove $\cosh^2 \theta - \sinh^2 \theta = 1$.

$D_x(\sinh x) = \cosh x$	$D_x(\cosh x) = \sinh x$
$D_x(\tanh x) = \operatorname{sech}^2 x$	$D_x(\coth x) = -\operatorname{csch}^2 x$
$D_x(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$	$D_x(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

EX 1

$D_x(\coth(4x)\sinh x) =$

EX 2

$\int \tanh x \ln(\cosh x)\, dx =$

EX 3

Verify this identity.

$$\tanh(x-y) = \frac{\tanh x - \tanh y}{1 - \tanh x \tanh y} \qquad \text{Hint: } \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Inverse Hyperbolic Functions

Let $\mathrm{y} = \sinh \mathrm{x} \Leftrightarrow \mathrm{x} = \sinh^{-1} \mathrm{y}$ (if the inverse exists).

Find $\mathrm{x} = \sinh^{-1} \mathrm{y}$.

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right) \qquad \cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)\ x \geq 1$$

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)\ x \in (-1,1) \quad \mathrm{sech}h^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)\ x \in (0,1]$$

How do we find the derivative of these functions?

$$D_x(\sinh^{-1} x) = D_x\left(\ln\left(x + \sqrt{x^2+1}\right)\right)$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}, \quad x > 1$$

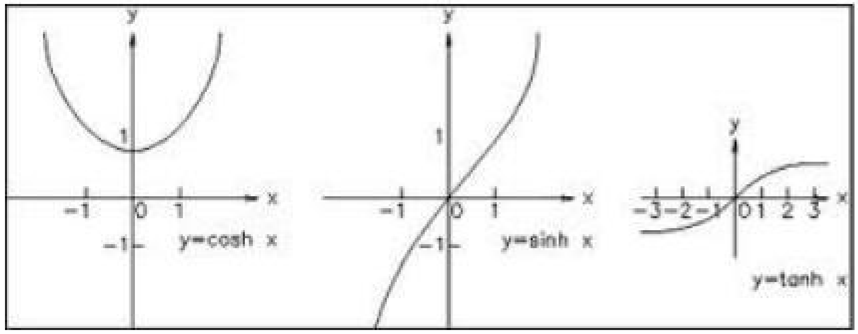
$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}, \quad x \in (-1,1)$$

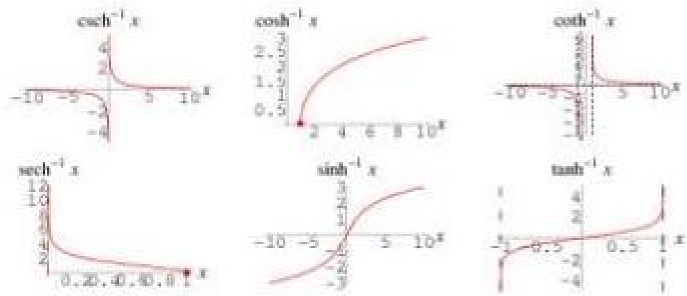
$$\frac{d}{dx}(\mathrm{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}, \quad x \in (0,1)$$

EX 4

Find y′.

$$y = x^2 \sinh^{-1}(x^5)$$


y=cosh x
y=sinh x
y=tanh x


csch⁻¹ x
cosh⁻¹ x
coth⁻¹ x
sech⁻¹ x
sinh⁻¹ x
tanh⁻¹ x