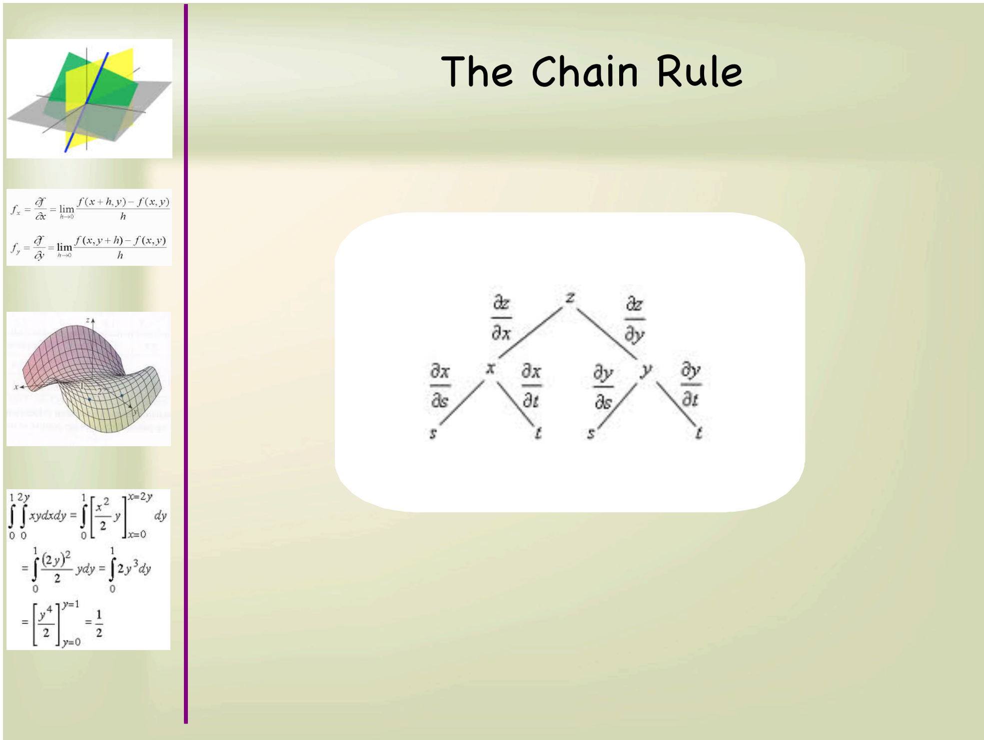


Recall: Chain rule for $y = f(g(x))$ is $y' = f'(g(x))g'(x) = \frac{df}{dg}\frac{dg}{dx}$.

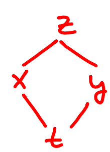
Chain Rules

Theorem

Let $x = x(t)$ and $y = y(t)$ be differentiable at t and let $z = f(x,y)$ be differentiable at $(x(t),y(t))$.

Then $z = f(x(t),y(t))$ is differentiable at t and

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} \Leftrightarrow \frac{dz}{dt} = \nabla f \cdot \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$$



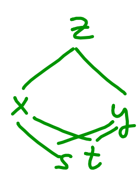
Theorem

Let $x = x(s,t)$ and $y = y(s,t)$ have first partial derivatives at $= \langle s, t \rangle$ and let $z = f(x,y)$ be differentiable at $(x(s,t), y(s,t))$.

Then z has first partial derivatives given by

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$



EX 1 Find $\frac{\partial w}{\partial t}$ given $w = x^2y - y^2x$, $x = \cos t$, $y = \sin t$.
Express the answer in terms of t.

EX 2 Find $\frac{\partial w}{\partial t}$ given $w = ln(x+y) - ln(x-y), \quad x=te^{s}, \; y = e^{st}$.

Express the answer in s and t .

EX 3 If $w = xy + x + y, \quad x = r + s + t \quad and\, y = rst,$

find $\left.\frac{\partial w}{\partial t}\right|_{r=1,\, s=-1,\, t=2}$.

EX 4 Sand is pouring onto a conical pile in such a way that at a certain instant, the height is 100 inches and increasing at 3 in/min. The base radius at that instant is 40 inches and increasing at 2 in/min. How fast is the volume increasing at that instant?

Implicit Differentiation

Let's go back to $y = f(x)$ and assume that instead of getting y as a function of x (explicitly), we have $F(x,y) = k$ for any constant, k (i.e. y is defined implicitly). Then, we just differentiated both sides with respect to x to get dy/dx .

For example:

$y^3 - 2xy + 3x = 4$

Apply the same ideas:

$xy^3 + sec(y+z) - zx^2 = 1$

EX 5 If $ye^{-x} + sin(x+z) + e^z = 5$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.