Math 2210 #17
Maxima and Minima

Recall from Calculus I:

1. Critical points (where $f'(x) = 0$ or DNE) are the candidates for where local min and max points can occur.
2. You can use the Second Derivative Test (SDT) to test whether a given critical point is a local min or max. SDT is not always conclusive.
3. Global max and min of a function on an interval can occur at a critical point in the interior of the interval or at the endpoints of the interval.

Extreme Values

1. f has a <u>global maximum</u> at a point (a, b) if $f(a, b) \geq f(x, y)$ for all (x, y) in the domain of f. f has a <u>local maximum</u> at a point (a, b) if $f(a, b)) \geq f(x, y)$ for all (x, y) near (a, b).
2. f has a <u>global minimum</u> at a point (a, b) if $f(a, b) \leq f(x, y)$ for all (x, y) in the domain of f. f has a <u>local minimum</u> at a point (a, b) if $f(a, b)) \leq f(x, y)$ for all (x, y) near (a, b).

Theorem (Critical Point)

Let f be defined on a set S containing (a, b). If $f(a, b)$ is an extreme value (max or min), then (a, b) must (be a critical point, i.e. either (a, b) is

a) a boundary point of S
b) a stationary point of S (where $\nabla f(a, b) = \vec{0}$, i.e. the tangent plane is horizontal)
c) a singular point of S (where f is not differentiable).

Fact: Critical points are candidate points for both global and local extrema.

Theorem (Max-Min Existence)

If f is continuous on a closed, bounded set S, then f attains both a global max value and a global min value there.

Second Partials Test Theorem

Suppose f(x, y) has continuous second partial derivatives in a neighborhood of (a, b) and $\nabla f(a, b) = \overline{0}$.

Let $D = D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - f_{xy}{}^2(a, b)$
then

1. If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local max.
2. If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local min.
3. If $D < 0$, then $f(a, b)$ is not an extreme value.
   ((a,b) is a saddle point.)
4. If $D = 0$ the test is inconclusive.

EX 1

For $f(x, y) = xy^2 - 6x^2 - 3y^2$, find all critical points, indicating whether each is a local min, a local max or saddle point.

EX 2

Find the global max and min values for

$$f(x, y) = x^2 - y^2 - 1 \text{ on}$$
$$S = \{(x, y) \mid x^2 + y^2 \leq 1\}$$

EX 3

Find the points where the global max and min occur for

$$f(x, y) = x^2 + y^2 \text{ on } S = \{(x, y) \mid x \in [-1,3], y \in [-1,4]\}$$

EX 4

Find the 3-D vector of length 9 with the largest possible sum of its components.