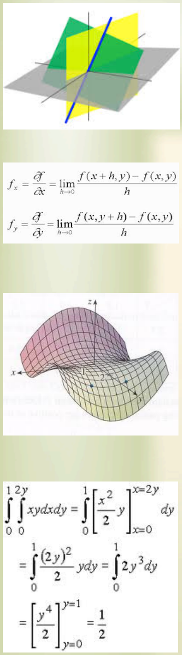
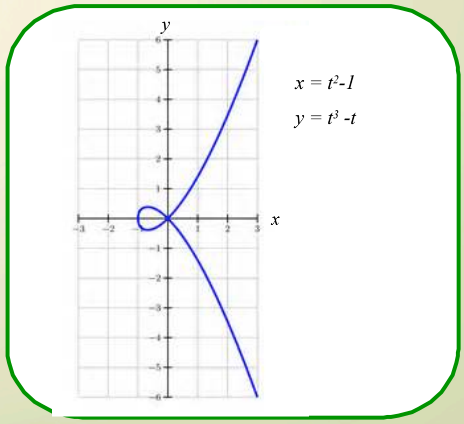


Parametric Representations of Plane Curves

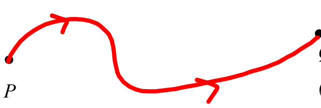


A plane curve is a 2-dimensional curve given by

$$x = f(t) \qquad y = g(t) \qquad t \in I$$

where f and g are continuous functions on the interval I, $[a,b]$.

t is the parameter. $\quad t \in [a,b]$



P

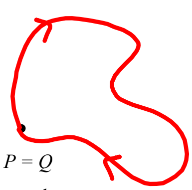
(x(a), y(a))

initial endpoint

Q

(x(b), y(b))

terminal endpoint

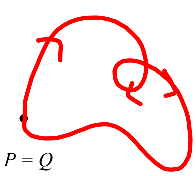


$P = Q$

simple,

closed

curve

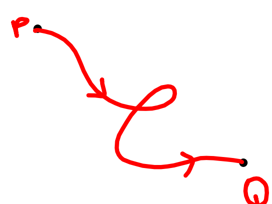


$P = Q$

not simple,

closed

curve



$P \neq Q$

not simple,

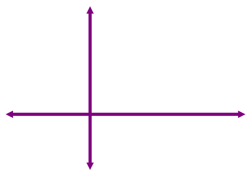
not closed

curve

It can be hard to recognize the shape of a curve when given parametically.
Sometimes it is possible to eliminate the parameter.

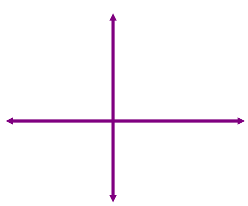
EX 1 Eliminate the parameter and sketch this curve.

$$x = t - 3 \qquad y = \sqrt{t} \qquad 0 \le t \le 8$$



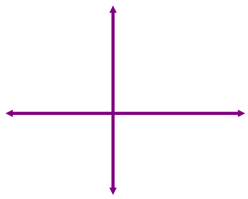
EX 2 Eliminate the parameter t, graph the curve and tell if it is simple and closed.

$$x = \sqrt{t-3} \qquad y = \sqrt{4-t} \qquad 3 \le t \le 4$$



EX 3 Eliminate the parameter θ, graph the curve and tell if it is simple and closed.

$$x = \sin\theta \qquad y = 2\cos^2(2\theta) \qquad \theta \in \Re$$



Theorem A

Let f and g be continuously differentiable with $f'(t) \neq 0$ on $t \in (\alpha, \beta)$.

Then the parametric equations $x = f(t)$ and $y = g(t)$

define y as a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \qquad \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} \quad where\ y' = \frac{dy}{dx}$$

EX 4 Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (without eliminating the parameter.)

a) $x = \sqrt{3}\theta^2 \qquad y = -\sqrt{3}\theta^3 \qquad \theta \neq 0$

b) $x = \dfrac{2}{1+t^2} \qquad y = \dfrac{2}{t(1+t^2)} \qquad t \neq 0$

Length of a curve: $L = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt$

EX 5 Find the length of the curve given by

$$x = \sin\theta - \theta\cos\theta \qquad \theta \in [\pi/4, \pi/2]$$
$$y = \cos\theta + \theta\sin\theta$$