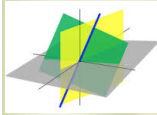
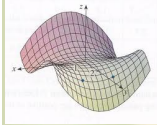


# Triple Integrals



$$f'_x = \frac{\partial}{\partial x} = \lim_{h \rightarrow 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$f'_y = \frac{\partial}{\partial y} = \lim_{h \rightarrow 0} \frac{f(x, y+h) - f(x, y)}{h}$$



$$\int_0^1 \int_0^{2y} xy \, dx \, dy = \int_0^1 \left[ \frac{x^2}{2} y \right]_{x=0}^{x=2y} dy$$

$$= \int_0^1 \frac{(2y)^2}{2} y \, dy = \int_0^1 2y^3 \, dy$$

$$= \left[ \frac{y^4}{2} \right]_{y=0}^{y=1} = \frac{1}{2}$$

$$\begin{aligned} \iiint_B 8xyz \, dV &= \int_1^2 \int_2^3 \int_0^4 8xyz \, dz \, dx \, dy \\ &= \int_1^2 \int_2^3 4xyz^2 \Big|_0^4 \, dx \, dy \\ &= \int_1^2 \int_2^3 4xy \, dx \, dy \\ &= \int_1^2 2x^2 y \Big|_2^3 \, dy \\ &= \int_1^2 10y \, dy = 15 \end{aligned}$$

## Triple Integrals

$$A = \int_a^b f(x) \, dx$$

Measures 2-D space (signed area) under a curve above the  $x$ -axis.

$$V = \iiint_S f(x, y) \, dA$$

Measures 3-D space (signed volume) under a surface above the  $xy$ -plane.

We predict that  $\iiint_S f(x, y, z) \, dV$  measures 4-D space (signed) under a "hyper" surface "above" the  $xyz$ -"hyper-plane".

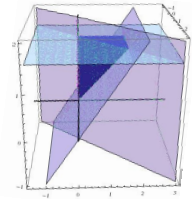
$$\iiint_S f(x, y, z) \, dV = \int_{a_1}^{a_2} \int_{\phi_1(x)}^{\phi_2(x)} \int_{\psi_1(x, y)}^{\psi_2(x, y)} f(x, y, z) \, dz \, dy \, dx$$

Note: We can't draw anything in 4-D, but we can draw the region  $S$  in 3-D (domain space is now 3-D).

EX 1 Write an iterated integral for  $\iiint_S (y+z+t)dV$

where  $S = \{(x,y,z) \mid x \in [0,1], y \in [2,5], z \in [1,4]\}$ .

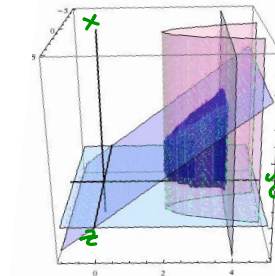
EX 2 Evaluate  $\int_0^{\pi/2} \int_0^z \int_0^y \sin(x+y+z) dx dy dz$ .



EX 3 Write an iterated integral for  $\iiint_S f(x, y, z) dV$

where  $S$  is the region in the first octant bounded by the surface  $z = 9 - x^2 - y^2$  and the coordinate planes.

EX 4 Find the volume of the solid in the first octant bounded by the hyperbolic cylinder  $y^2 - 64z^2 = 4$  and the plane  $y = x$  and  $y = 4$ .



EX 5 Find the volume of the tetrahedron with vertices at  $(0,0,0)$ ,  $(0,0,3)$ ,  $(0,4,0)$ , and  $(2,0,0)$ .