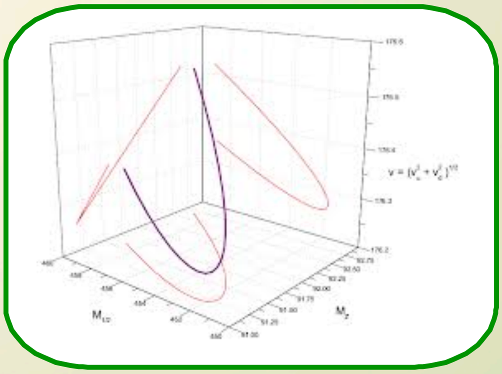
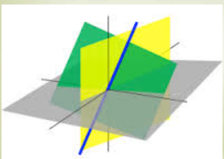
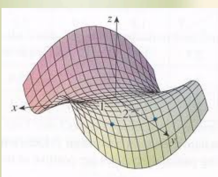
Cartesian Coordinates
in
3-Space





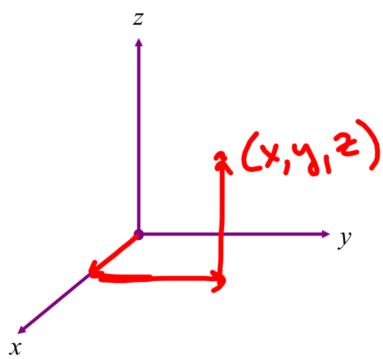
$$f_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

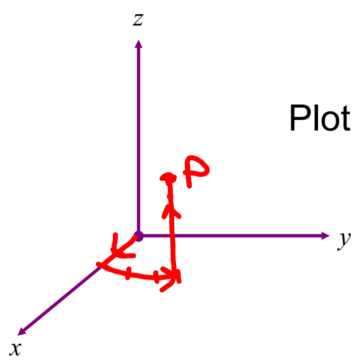
$$f_y = \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$



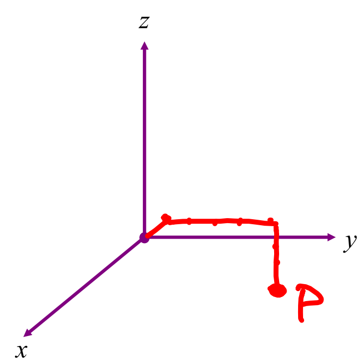
$$\int_0^1 \int_0^{2y} xy\,dx\,dy = \int_0^1 \left[\frac{x^2}{2} y\right]_{x=0}^{x=2y} dy$$
$$= \int_0^1 \frac{(2y)^2}{2} y\,dy = \int_0^1 2y^3\,dy$$
$$= \left[\frac{y^4}{2}\right]_{y=0}^{y=1} = \frac{1}{2}$$

A point in 3-space is given by an ordered triple (x,y,z).



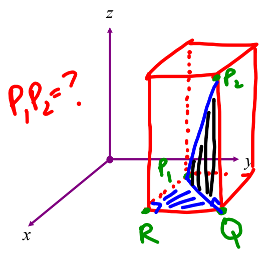


Plot $(2,3,4)$



Plot $(-1,4,-3)$

Distance Formula $d^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$



ΔP_1P_2Q and ΔP_1QR are both right triangles.

$\Rightarrow$ can use Pythagorean Thm

① $|P_1Q|^2 + |QP_2|^2 = |P_1P_2|^2$

and ② $|P_1R|^2 + |RQ|^2 = |P_1Q|^2$

note: P_1Q means length of line segment from P_1 to Q

$\Rightarrow$ sub ② into ①

$$|P_1R|^2 + |RQ|^2 + |QP_2|^2 = |P_1P_2|^2 \qquad (*)$$

if P_1 is the pt (x_1, y_1, z_1) and P_2 is (x_2, y_2, z_2),

then $Q\ (x_2, y_2, z_1)$

$R\ (x_2, y_1, z_1)$

$|P_1R|^2 = (x_1 - x_2)^2$ (since y and z coords are the same)

$|RQ|^2 = (y_1 - y_2) + (z_1 - z_2)^2$ (since these 2 pts are on same plane, so this is 2-d length formula)

$(*)$ $\Rightarrow |P_1P_2|^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$

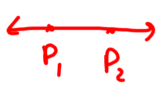
$$\Rightarrow \boxed{P_1P_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}}$$

$P_1 = (x_1, y_1, z_1)$

$P_2 = (x_2, y_2, z_2)$

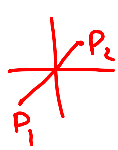
Summary

1-d:



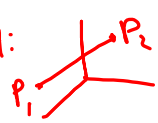
$P_1P_2 = |x_1 - x_2| = \sqrt{(x_1 - x_2)^2}$

2-d:



$P_1P_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

3-d:



$P_1P_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

EX 1 Show that these points are vertices of an equilateral triangle.

$(4,5,3),\ (1,7,4),\ (2,4,6)$

A B C

show $AB = BC = AC$

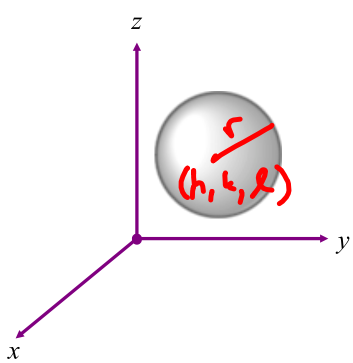
$AB = \sqrt{(4-1)^2+(5-7)^2+(3-4)^2} = \sqrt{9+4+1} = \sqrt{14}$

$BC = \sqrt{(1-2)^2+(7-4)^2+(4-6)^2} = \sqrt{1+9+4} = \sqrt{14}$

$AC = \sqrt{(4-2)^2+(5-4)^2+(3-6)^2} = \sqrt{4+1+9} = \sqrt{14}$ ✓

Spheres All points (x,y,z) on a sphere are a fixed distance, r from the center.

$$r = \sqrt{(x-h)^2 + (y-k)^2 + (z-l)^2}$$



So the equation of a sphere with radius r and center (h,k,l) is $\boxed{r^2 = (x-h)^2 + (y-k)^2 + (z-l)^2}$

Midpoint of the segment (x_1,y_1,z_1) and (x_2,y_2,z_2)

$$\boxed{m = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)}$$

Ex 2

a) Find the center and radius of this sphere.

$$x^2 + y^2 + z^2 + 2x - 6y - 10z + 34 = 0$$

we want in this form $(x-h)^2 + (y-k)^2 + (z-\ell)^2 = r^2$

$$(x^2 + 2x + 1) + (y^2 - 6y + 9) + (z^2 - 10z + 25)$$

$$= -34 + 1 + 9 + 25$$

$$(x+1)^2 + (y-3)^2 + (z-5)^2 = 1$$

$\Rightarrow$ $\boxed{\text{center } (-1, 3, 5) \quad r = 1}$

b) Find the equation of the sphere that has a diameter from $(-4, 2, 1)$ to $(8, 3, 6)$.

center of sphere is midpt of any diameter

$$\text{midpt} = \left(\frac{-4+8}{2}, \frac{2+3}{2}, \frac{1+6}{2}\right) = \left(2, \frac{5}{2}, \frac{7}{2}\right) \text{ center}$$

$$r = \frac{d}{2} = \frac{1}{2}\sqrt{(-4-8)^2 + (2-3)^2 + (1-6)^2} = \frac{1}{2}\sqrt{144 + 1 + 25}$$

$$= \frac{1}{2}\sqrt{170}$$

sphere: $(x-2)^2 + \left(y - \frac{5}{2}\right)^2 + \left(z - \frac{7}{2}\right)^2 = \left(\frac{1}{2}\sqrt{170}\right)^2$

$$\boxed{(x-2)^2 + \left(y - \frac{5}{2}\right)^2 + \left(z - \frac{7}{2}\right)^2 = \frac{85}{2}}$$

<u>Linear equations</u> in 3-space

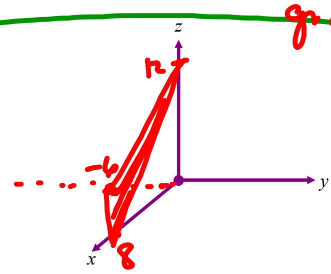
$$Ax + By + Cz = D$$

A, B, C, D are constants

(eqn of a plane in 3-d)

1-d:	linear eqn	ex $3x=5$ graphs into pt
2-d:	" "	ex $3x+2y=5$ graphs into line
3-d:	" "	ex $3x+2y+4z=5$ graphs into a plane

EX 3 Graph $3x - 4y + 2z = 24$.



graph the pts on coordinate axes:

$x=y=0$: $2z=24 \Rightarrow z=12$ $(0,0,12)$

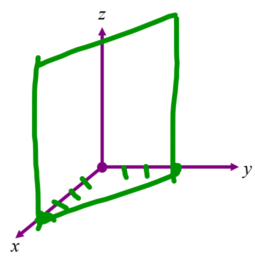
$x=z=0$: $-4y=24 \Rightarrow y=-6$ $(0,-6,0)$

$y=z=0$: $3x=24 \Rightarrow x=8$ $(8,0,0)$

EX 4 Graph $3x + 4y = 12$.

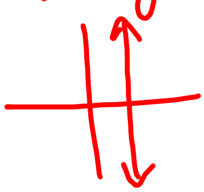
(note: this is a linear eqn.)

this graphs into a plane in 3-d



$x = 0$,
$4y = 12$
$y = 3$
$(0, 3, z)$

in 2-d, to graph $x = 3$



(a line)

(a plane that is $\parallel$ to z-axis)

$y = 0$
$3x = 12$
$x = 4$
$(4, 0, z)$