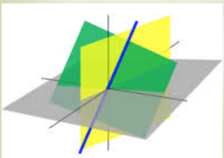
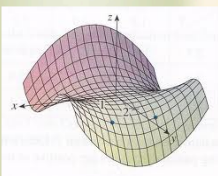


$$f_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$f_y = \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

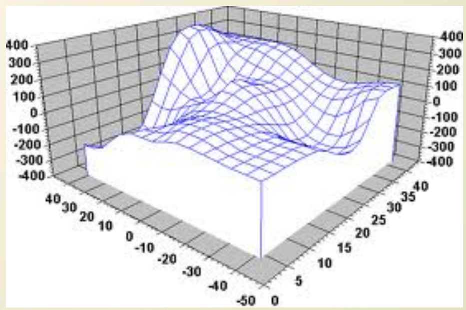


$$\int_0^1 \int_0^{2y} xy\,dx\,dy = \int_0^1 \left[\frac{x^2}{2} y\right]_{x=0}^{x=2y} dy$$

$$= \int_0^1 \frac{(2y)^2}{2} y\,dy = \int_0^1 2y^3\,dy$$

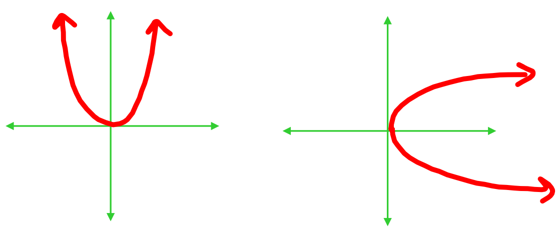
$$= \left[\frac{y^4}{2}\right]_{y=0}^{y=1} = \frac{1}{2}$$

Surfaces in Three-Space



Quick Review of the Conic Sections

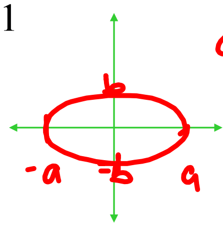
a) Parabola $y = x^2$ $x = y^2$



b) Ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

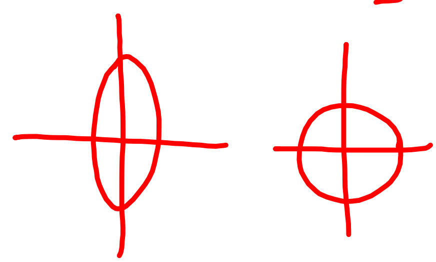
if $y = 0$, $x = \pm a$

if $x = 0$, $y = \pm b$

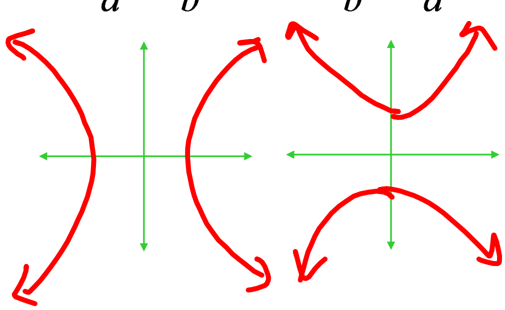


$a > b$

$a < b$ $a = b$



c) Hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$



Surfaces in Three-Space

The graph of a 3-variable equation which can be written in the form $F(x,y,z) = 0$ or sometimes $z = f(x,y)$ (if you can solve for z) is a surface in 3D. One technique for graphing them is to graph cross-sections (intersections of the surface with well-chosen planes) and/or traces (intersections of the surface with the coordinate planes).

We already know of two surfaces:

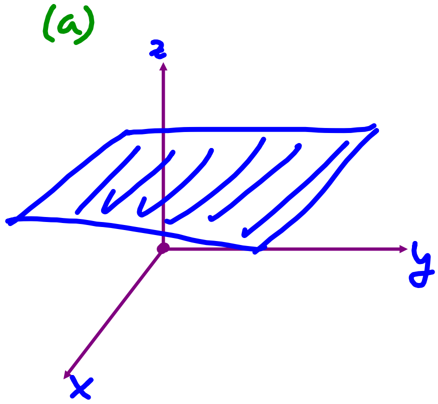
$F(x,y,z)=0$ (implicit)

a) plane $Ax + By + Cz = D$

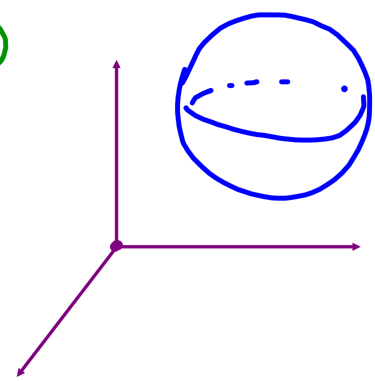
$z=f(x,y)$ (explicit)

b) sphere $(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$

(a)



(b)

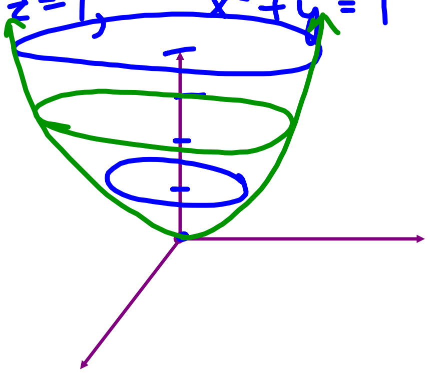


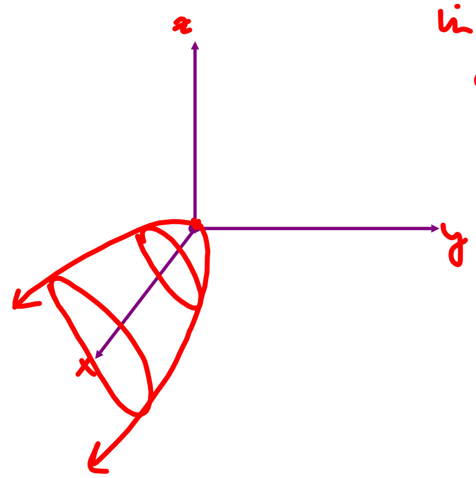
EX 1 Sketch a graph of $z = x^2 + y^2$ *and* $x = y^2 + z^2$.

(shape will be same as in left drawing)

if $z = 0$, $x = 0$, $y = 0$

if $z = 1$, $x^2 + y^2 = 1$





if $z = 4$, $x^2 + y^2 = 4$

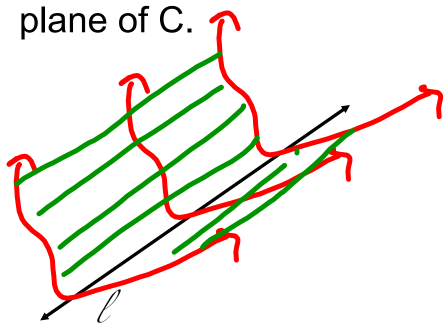
cross-sections:

in xy-plane, circles

in xz-plane, parabolas

in yz-plane, parabolas

A <u>cylinder</u> is the set of all points on lines parallel to ℓ that intersect C where C is a plane curve and ℓ is a line intersecting C, but not in the plane of C.



A <u>Quadric Surface</u> is a 3D surface whose equation is of the second degree.

The general equation is

$$\boxed{Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0,}$$

given that $A^2 + B^2 + C^2 \neq 0$.

With rotation and translation, these possibilities can be reduced to two distinct types.

note: a plane is not a quadric surface

1) $Ax^2 + By^2 + Cz^2 + J = 0$

all three quadratic terms; no linear terms

2) $Ax^2 + By^2 + Iz = 0$

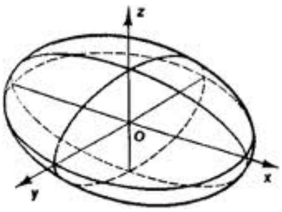
only 2 quadratic plus one linear term

Basic Quadric Surfaces

(on this page, all are type (1))

ELLIPSOID

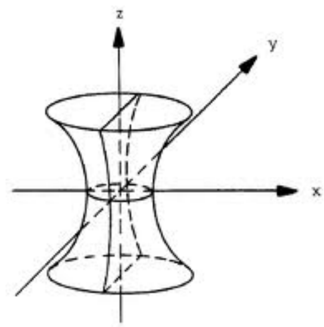
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$



note: a sphere is an ellipsoid (w/ a=b=c)

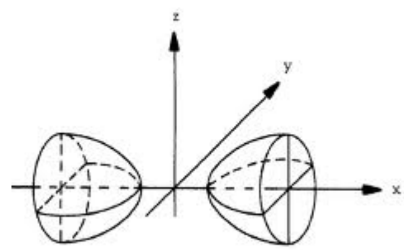
HYPERBOLOID OF ONE SHEET

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\underbrace{\frac{z^2}{c^2}}_{\text{axis}}=1$$



HYPERBOLOID OF TWO SHEETS

$$\underbrace{\frac{x^2}{a^2}}_{\text{axis}}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

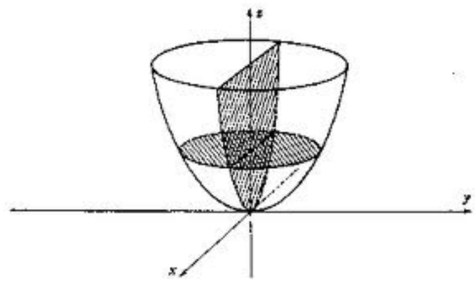


type (2)

ELLIPTIC PARABOLOID

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

axis

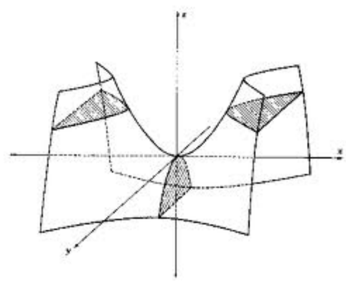


type (2)

HYPERBOLIC PARABOLOID

$$z = \frac{y^2}{b^2} - \frac{x^2}{a^2}$$

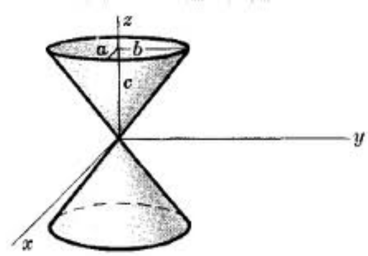
(looks like saddle or Pringles potato chip)



type (1)

ELLIPTIC CONE

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$



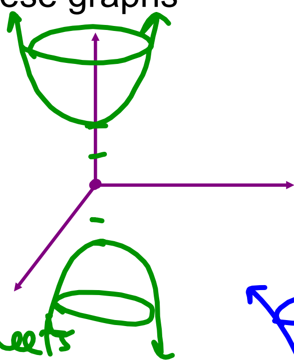
EX 2 Name and sketch these graphs

a) $9x^2 + y^2 - z^2 = -4$

(no linear terms)

$$-\frac{9x^2}{4} - \frac{y^2}{4} + \frac{z^2}{4} = 1$$

hyperboloid of 2 sheets

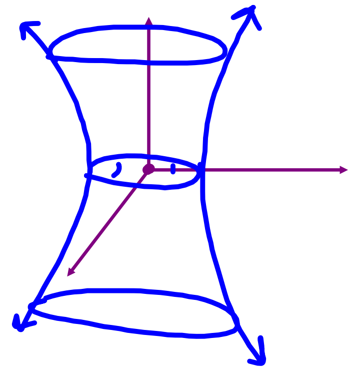


b) $9x^2 + y^2 - z^2 = 4$

(no linear terms)

$$\frac{9x^2}{4} + \frac{y^2}{4} - \frac{z^2}{4} = 1$$

hyperboloid of one sheet about z-axis



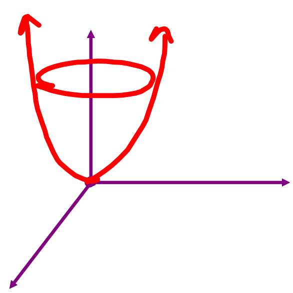
c) $x^2 + 4y^2 - z = 0$

(2 quadratic terms; one linear term)

about z-axis

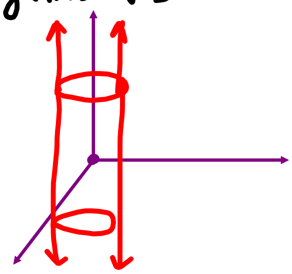
$$z = x^2 + 4y^2$$

elliptic paraboloid



note: these are all cylinders

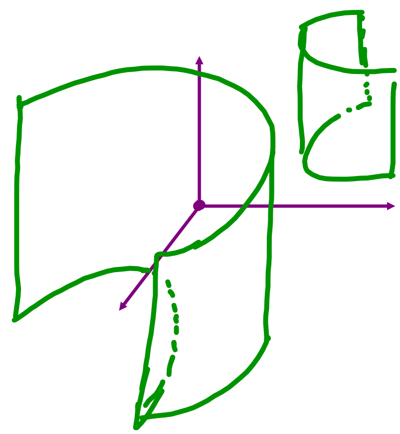
d) $x^2 + y^2 = 1$



(about z-axis)

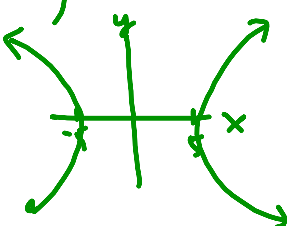
(all cross-sections for planes || to xy-plane are unit circles)

e) $x^2 - y^2 = 25$



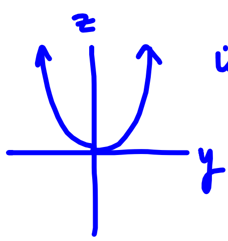
(about z-axis)

in 2-d $x^2 - y^2 = 25$ is a hyperbola



f) $z = y^2$

(about x-axis)



in 2-d