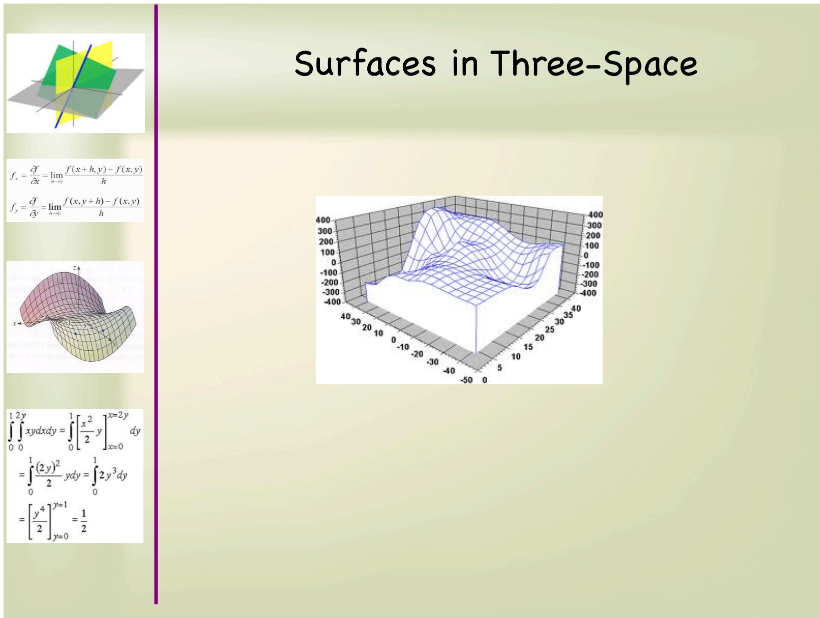
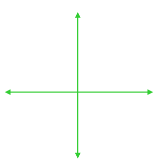
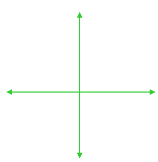
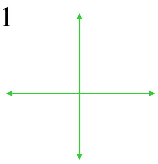


Quick Review of the Conic Sections

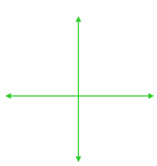
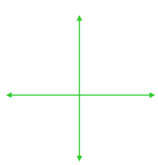
a) Parabola $y = x^2$ $x = y^2$

b) Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



c) Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

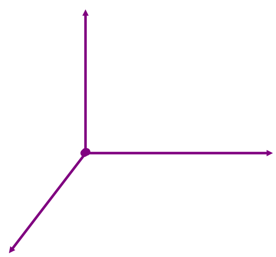
Surfaces in Three-Space

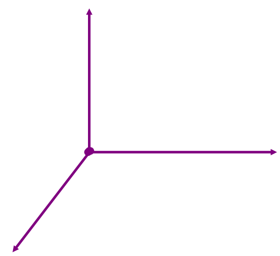
The graph of a 3-variable equation which can be written in the form $F(x,y,z) = 0$ or sometimes $z = f(x,y)$ (if you can solve for z) is a surface in 3D. One technique for graphing them is to graph cross-sections (intersections of the surface with well-chosen planes) and/or traces (intersections of the surface with the coordinate planes).

We already know of two surfaces:

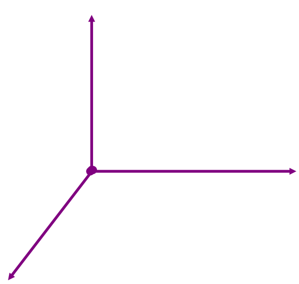
a) plane $Ax + By + Cz = D$

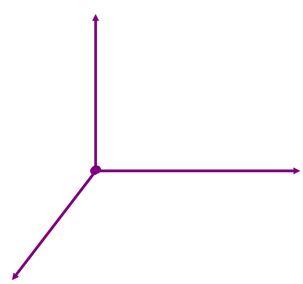
b) sphere $(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$



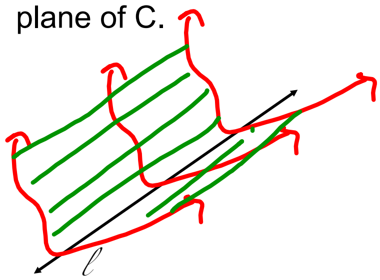


EX 1 Sketch a graph of $z = x^2 + y^2$ *and* $x = y^2 + z^2$.





A cylinder is the set of all points on lines parallel to ℓ that intersect C where C is a plane curve and ℓ is a line intersecting C, but not in the plane of C.



A Quadric Surface is a 3D surface whose equation is of the second degree.

The general equation is

$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$,

given that $A^2 + B^2 + C^2 \neq 0$.

With rotation and translation, these possibilities can be reduced to two distinct types.

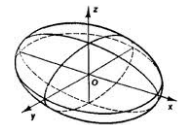
1) $Ax^2 + By^2 + Cz^2 + J = 0$

2) $Ax^2 + By^2 + Iz = 0$

Basic Quadric Surfaces

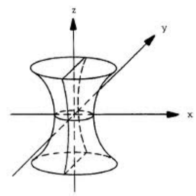
ELLIPSOID

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$



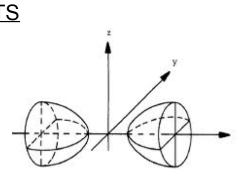
HYPERBOLOID OF ONE SHEET

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$



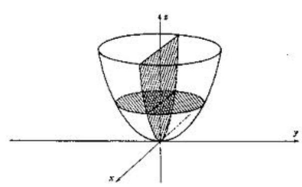
HYPERBOLOID OF TWO SHEETS

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$



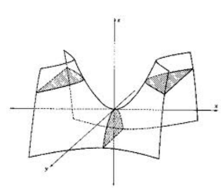
ELLIPTIC PARABOLOID

$$z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$$



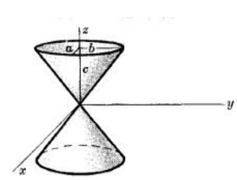
HYPERBOLIC PARABOLOID

$$z=\frac{y^2}{b^2}-\frac{x^2}{a^2}$$



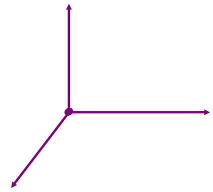
ELLIPTIC CONE

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

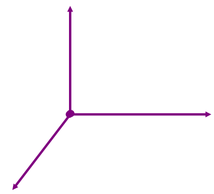


EX 2 Name and sketch these graphs

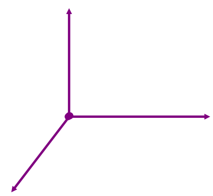
a) $9x^2 + y^2 - z^2 = -4$



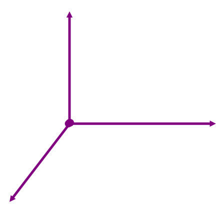
b) $9x^2 + y^2 - z^2 = 4$



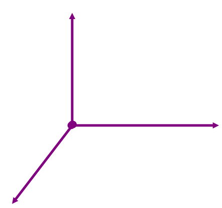
c) $x^2 + 4y^2 - z = 0$



d) $x^2 + y^2 = 1$



e) $x^2 - y^2 = 25$



f) $z = y^2$