MATH CIRCLE CONTEST I
October 27, 2004

1. NO COSTUME? YOU CAN ALWAYS COUNT ON HALLOWEEN.

Suppose on Halloween that you have an bag of candy which contains infinitely many pieces of each of three kinds: gumdrops, lemon sours, and licorice. Being quite generous, you give a local goblin ten pieces of candy. How many possible candy combinations are there?

2. ELECTION DAY

A citizen is to vote yes or no on each of eight issues this election. She is allowed to abstain from voting on some of the issues, but she must vote on at least five. How many possible ways can she vote? (Assume no hanging chads.)

3. FERMAT MOD ELEVEN

Find all solutions to

$$x^9 + y^9 = 3 \mod 11$$

for integers x and y between 0 and 10. For instance $x = 1$ and $y = 6$ is a such solution since: $1^9 = 1 \mod 11$; $6^9 = 10077696$ which is 2 mod 11; and hence $1^9 + 6^9 = 1 + 2 = 3 \mod 11$. (Hint: recall what Fermat's Little Theorem tells you about $a^p \mod p$.)

4. BINOMIAL CONTORTIONS (THAT'S A HINT!)

Find the constant term (i.e. the one with no powers of x) in

$$\left(x^2 - \frac{2}{x}\right)^{24}.$$

(You need not reduce your answer to an actual numerical value; leaving it in a form analogous to $(100!)/3^7$ is acceptable.)