MATH CIRCLE FINALE
May 3, 2006

1. When you multiply out
$$(xyz - x^2 - y^2 + z - xz + y + 2)^5$$
what is the sum of all the coefficients?

2. My watch runs 1 second fast per hour. Eric's runs 1.5 seconds slow per hour. Right now they both read the same time (which is the correct time).

    (a) When will our watches read the same time again?

    (b) When will our watches read the same *correct* time again?

3. Suppose you toss five fair six-sided dice. What is the probability that at least one 4, at least one 5, and at least one 6 appear in the toss?

4. Suppose N is a number all of whose digits are the same. Suppose further that N is the square of an integer; that is, suppose there is an integer m such that $N = m^2$. Determine all possible values of N. (For example $N = 9$ is one possibility: it has only one digit (so the condition that the digits be the same holds) and it is of course the square of $m = 3$.)