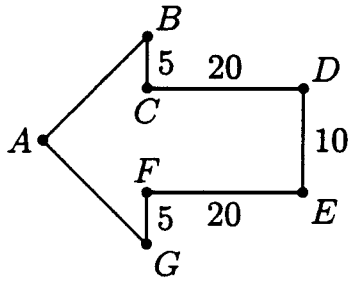
Math Circle Contest - Grade 10

February 5, 2003

1. In the arrow-shaped polygon [see figure], the angles at vertices A, C, D, E and F are right angles, $BC = FG = 5$, $CD = FE = 20$, $DE = 10$, and $AB = AG$. The area of the polygon is closest to



(A) 288 **(B)** 291 **(C)** 294

(D) 297 **(E)** 300

2. The integers 1, 2, 3, 4 are arranged in a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. The probability that the determinant, $ad - bc$, is even is

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$ (e) $\frac{4}{5}$

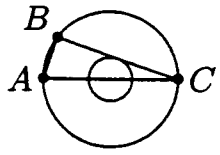
3. Each day Walter gets \$3 for doing his chores or \$5 for doing them exceptionally well. After 10 days of doing his chores daily, Walter has received a total of \$36. On how many days did Walter do them exceptionally well?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

4. What is the sum of the digits of the decimal form of the product $2^{1999} \cdot 5^{2001}$?

(A) 2 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 10

5. The ratio of the radii of two concentric circles is $1:3$. If $\overline{AC}$ is a diameter of the larger circle, $\overline{BC}$ is a chord of the larger circle that is tangent to the smaller circle, and $AB = 12$, then the radius of the larger circle is



(A) 13 **(B)** 18 **(C)** 21 **(D)** 24 **(E)** 26

6. In how many ways can 5 dollars be paid in dimes and quarters?
(a) 25 (b) 24 (c) 10 (d) 11 (e) None of these

7. Renzo rolls a fair regular octahedral die marked with the numbers 1 through 8. Then Nick rolls a fair six-sided die. What is the probability that the product of the two rolls is a multiple of 3?
(a) $\frac{1}{12}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{7}{12}$ (e) $\frac{2}{3}$

8. **Let f be a linear function with the properties that $f(1) \leq f(2)$, $f(3) \geq f(4)$, and $f(5) = 5$. Which of the following statements is true?**

(A) $f(0) < 0$ **(B)** $f(0) = 0$ **(C)** $f(1) < f(0) < f(-1)$
(D) $f(0) = 5$ **(E)** $f(0) > 5$

9. For how many values of k is 12^{12} the least common multiple of the positive integers 6^6, 8^8, and k?

10. Find all positive integers x and y such that $x^2 = y^2 + 77$.

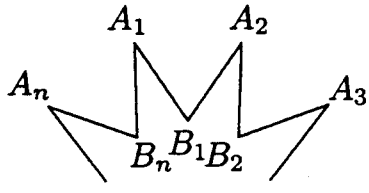
11. In how many ways can 7 people be divided into two teams, with each team having at least one member?

(a) 72 (b) 32 (c) 144 (d) 48 (e) 63

12. Point P is 9 units from the center of a circle of radius 15. How many different chords of the circle contain P and have integer lengths?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 29

13. Part of an "n-pointed regular star" is shown. It is a simple closed polygon in which all $2n$ edges are congruent, angles A_1, A_2, ..., A_n are congruent and angles B_1, B_2, ..., B_n are congruent. If the acute angle at A_1 is $10°$ less than the acute angle at B_1, then $n =$



(A) 12 **(B)** 18 **(C)** 24 **(D)** 36 **(E)** 60