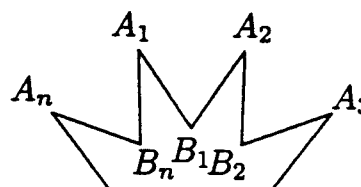


Math Circle Contest - Grade 12

February 5, 2003

1. Part of an “ n -pointed regular star” is shown. It is a simple closed polygon in which all $2n$ edges are congruent, angles A_1, A_2, \dots, A_n are congruent and angles B_1, B_2, \dots, B_n are congruent. If the acute angle at A_1 is 10° less than the acute angle at B_1 , then $n =$



- (A) 12 (B) 18 (C) 24 (D) 36 (E) 60

2. In how many ways can 7 people be divided into two teams, with each team having at least one member?

- (a) 72 (b) 32 (c) 144 (d) 48 (e) 63

3. Point P is 9 units from the center of a circle of radius 15. How many different chords of the circle contain P and have integer lengths?

- (A) 11 (B) 12 (C) 13 (D) 14 (E) 29

4. Find all positive integers x and y such that $x^2 = y^2 + 77$.

5. Let f be a linear function with the properties that $f(1) \leq f(2)$, $f(3) \geq f(4)$, and $f(5) = 5$. Which of the following statements is true?
- (A) $f(0) < 0$ (B) $f(0) = 0$ (C) $f(1) < f(0) < f(-1)$
(D) $f(0) = 5$ (E) $f(0) > 5$

6. Renzo rolls a fair regular octahedral die marked with the numbers 1 through 8. Then Nick rolls a fair six-sided die. What is the probability that the product of the two rolls is a multiple of 3?

- (a) $\frac{1}{12}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{7}{12}$ (e) $\frac{2}{3}$

7. For how many values of k is 12^{12} the least common multiple of the positive integers 6^6 , 8^8 , and k ?

8. A box contains 3 blue balls and 4 red balls. One by one, balls are drawn at random from the box and not replaced. Find the probability that it will take more than four draws until the third blue ball appears.

- (a) $\frac{4}{35}$ (b) $\frac{4}{5}$ (c) $\frac{6}{7}$ (d) $\frac{31}{35}$ (e) $\frac{32}{35}$

9. The sequence a_1, a_2, a_3, \dots satisfies $a_1 = 19$, $a_9 = 99$, and, for all $n \geq 3$, a_n is the arithmetic mean of the first $n - 1$ terms. Find a_2 .

- (A) 29 (B) 59 (C) 79 (D) 99 (E) 179

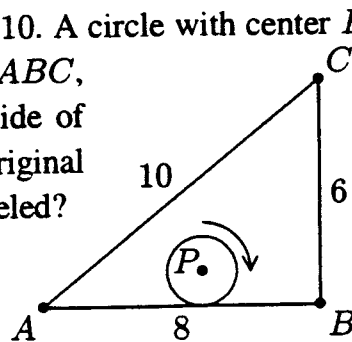
10. If $\{a_1, a_2, \dots\}$ is a set of positive integers (not necessarily distinct) whose sum is 2003, what is the maximum their product could be?

11. There are 323 farms in Box Elder county that have at least one of horses, cows, and sheep. If 224 have horses, 85 have cows, 57 have sheep, and 18 farms have all three types of animals, how many farms have exactly two of these types of animals?

- (a) 43 (b) 7 (c) 18 (d) 14 (e) 61

12. The sides of $\triangle ABC$ have lengths 6, 8 and 10. A circle with center P and radius 1 rolls around the inside of $\triangle ABC$, always remaining tangent to at least one side of the triangle. When P first returns to its original position, through what distance has P traveled?

- (A) 10 (B) 12 (C) 14
(D) 15 (E) 17



13. How many triangles with positive area are there whose vertices are points in the xy -plane whose coordinates are integers (x, y) satisfying $1 \leq x \leq 4$ and $1 \leq y \leq 4$?

- (A) 496 (B) 500 (C) 512 (D) 516 (E) 560