MATH CIRCLE CONTEST #1
**Revised!**
DUE NOVEMBER 14, 2001

$1^1$. A "crocodile" is a new chess-piece whose move consists of a jump of one cell horizontally or vertically, and then a jump of $N$ cells in the perpendicular direction. (So when $N = 2$, a crocodile is the usual knight in chess.) For what $N$ can the crocodile move from any cell to any other cell on an *infinite* chess board.

2. Consider a Young diagram and view its cells as boxes whose edges are length 1. Assign to each cell of the diagram a height which is a positive integer, so that the height above any cell is greater than or equal to both the height above the cell immediately to the East and the height above the cell immediately to the South. View this as a three dimensional figure which is built up of cubes whose edge is length 1. Color the figure so that each cube is either black or white, and so that adjacent cubes have different colors. Suppose that after coloring, the figure has an equal number of black and white cubes. Prove or disprove: the figure can be built from identical blocks whose dimensions are $1 \times 1 \times 2$.

3. Suppose that there are $N$ people in a room and each writes his or her name on a card. The cards are put in a box, the box is shaken, and then each person draws a card, without looking, from the box. Let $P(n, k)$ (for $0 \le k \le N$) be the probability that exactly $k$ people get their own card. Find an expression for $P(n, k)$ and for the expected value $E(N)$ defined as

$$E(N) = \sum_{k=0}^{N} P(N, k)k.$$

($E(N)$ is expected number of people who get their own card back.)

4. How many arrangements of $\{1, 2, 3, 4, 5, 6\}$ are there such that at least one of the following statements is true:

(1) 1 is to the left of 2;
(2) 2 is to the left of 3; or
(3) 3 is to the left of 4.

For example, 1 4 3 2 6 5 is such an arrangement since 1 is to the left of 2.

---

[1]Courtesy of Zvezdalina Stankova-Frenkel