MATH CIRCLE CONTEST II
December 14, 2005

1. MUSICAL CHAIRS, PART 1

Five people are seated at a table with five chairs. They are asked to get up and move to a new chair. How many new seating arrangements are possible?

2. MAKE HEADS OR TAILS OF IT, PART 1

Consider the following game played with seven pennies arranged in a row. The allowable moves are as follows: if a head appears in the ith position, a played may flip each penny adjacent to the ith position from head to tails, or vice versa, while still leaving a heads in the ith position. For instance, we have the following allowable move using the heads that occupies the third position (as indicated in boldface):

T H **H** T T T T

moves to

T T **H** H T T T.

Similarly,

H H T H T T T

moves to

H T T H T T T

Prove or disprove: HTTTTTT can be transformed to TTTTTTH.

3. POLITICAL SCIENCE (FOR MATHEMATICIANS)

A town meeting devoted to environmental issues is attended by Republicans, Democrats, and Independents. A number of participants wear green badges (indicating their pro-environment position). It transpires that there are an equal number of Republicans and Democrats, and that every Independent wears a green badge. Moreover there are twice as many green badges worn by Democrats than Republicans. If 501 people attend the meeting and there are 100 green badges, 21 of which are worn by Republicans, how many Democrats are in attendance?

4. MUSICAL CHAIRS, PART 2

This time five people are seated at a table with *seven* chairs. They are asked to get up and move to a new chair. How many new seating arrangements are possible?

5. make heads or tails of it, part 2

Retain the setting of Problem 2. Prove or disprove: THTTTTT can be transformed to TTHTTTT.