MATH CIRCLE CONTEST III

January 30, 2002

SWAPPING GAMES, PART 1

Consider the following game. The numbers $1, \ldots, 16$ are initially arranged in order. An allowable move consists of swapping the numbers in position i and $i+3$ and simultaneously swapping those in the $i+1$ and $i+2$ position; here $1 \leq i \leq 13$. For instance, if $i = 3$, the move takes the initial configuration

$$1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16$$

to

$$1 \ 2 \ 6 \ 5 \ 4 \ 3 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16$$

Prove or disprove: every sequence of the numbers $1, \ldots, 16$ can be obtained from the initial configuration through a series of allowable moves.

SWAPPING GAMES, PART 2

The situation similar to the first problem, but there are now only the numbers $1, \ldots, 8$ and the moves are different. An allowable move now consists of swapping the number in the i position with the one in the $i+1$ position; here $1 \leq i \leq 7$. For instance if $i = 3$, the move takes

$$1\ 2\ 3\ 4\ 5\ 6\ 7\ 8$$

to

$$1\ 2\ 4\ 3\ 5\ 6\ 7\ 8.$$

Michelle the mathematician finds that she can move from the initial configuration

$$1\ 2\ 3\ 4\ 5\ 6\ 7\ 8$$

to

$$8\ 7\ 6\ 1\ 3\ 4\ 5\ 2$$

in 25 moves. Prove or disprove: there is no shorter sequence of allowable moves taking the initial configuration to the one given above.

A waiter is taking dessert orders from a party of 15. The diners may choose among 8 different desserts. How many different dessert orders are possible provided at least one of every dessert gets ordered?