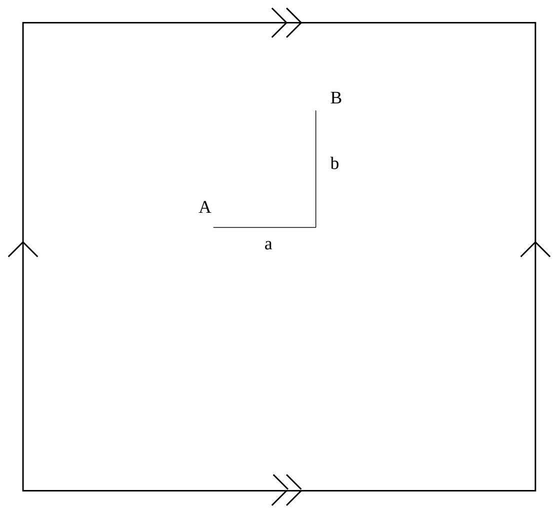
MATH CIRCLE CONTEST IV

March 27, 2002

LINES ON THE TORUS

Consider a torus obtained by identifying the edges of a 10-by-10 square as indicated below. Fix a point A on a torus, and let B be the point indicated below. Suppose an ant begins at point A and walks in a straight line through B and continues walking on the torus.



(a) Let $a = 2$ and $b = 5$. Prove or disprove: the ant's path will eventually cross itself.

(b) Now let a and b be arbitrary real numbers. Prove or disprove: the ant's path will eventually cross itself.

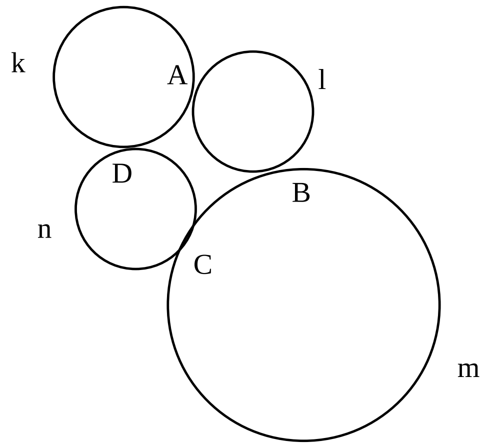
SHREDDING A MÖBIUS BAND

Consider a Möbius band of constant width 1 inch. Draw a line segment (one inch long) perpendicular to the edges.

(a) Place five scissors along the line segment at respective distances of 1/6, 2/6, 3/6, 4/6 and 5/6 of an inch from an edge. With the four scissors cut parallel to the edges until it is not possible to cut anymore. Describe the resulting object.

(b) Repeat (a) with six scissors at distances 1/7, 2/7, 3/7, 4/7, 5/7 and 6/7.

Circles k, l, m, and n are arranged so that k is tangent to l at the point A, l is tangent to m at B, m is tangent to n at C, and n is tangent to k at D. See the figure below.



Prove that A, B, C and D are either collinear or concyclic.