

MATH CIRCLE CONTEST I
October 25, 2006

1(A). TEMPLE SQUARE TO THE CITY LIBRARY, PART ONE

As everyone knows, the streets of Salt Lake City are laid out on a perfect grid with two-way streets running North-South and East-West each block. (This isn't quite true, but for the purposes of this problem assume it is indeed the case!) Suppose you start at Temple Square (which is 0 South and 0 East) and want to drop off a book at the city library drop box (at 5th South and 5th East) before returning to Temple Square. The catch is that you want to traverse each block in the intermediate 5-by-5 grid exactly once. Either find such a path or prove that no such path exists.

1(B). TEMPLE SQUARE TO THE CITY LIBRARY, PART TWO

As part of Mayor Rocky Anderson's sidewalk replacement project, it turns out that the blocks on South Temple (which is 0 South) and 5th South between 1st and 2nd East and between 3rd and 4th East are closed. Meanwhile the blocks on State Street (which is 0 East) and 5th East between 1st and 2nd South and between 3rd and 4th South are closed. Can you make it from Temple Square to the library drop box traversing each open block exactly once? Either find such a path or prove that no such path exists.

2. BREAKING EVEN AT HOMECOMING

At the West High Homecoming Party, 123 students arrive to show their school spirit. Show that at least one person at the party has an even number of friends in attendance. (For the purposes of this problem, assume that friendship is mutual. That is, if A is B's friend, then B is also A's friend.)

3(A). PAINTED SQUARES, PART ONE

Suppose you are given a collection of 1-by-1 squares and four cans of paint (red, blue, yellow, and green). Given such a square, you are allowed to paint each edge one color in such a way that the four edges are each painted a different color. You are also allowed to glue any two painted square together along two edges of the same color. Using this procedure, you want to assemble a 5-by-7 rectangle so that each edge of the resulting rectangle is painted a single color and so that each of the four colors appears on one of the edges of the rectangle. Can you do it? Either provide a coloring and gluing scheme or prove that no such exists.

3(B). PAINTED SQUARES, PART TWO

Repeat the previous problem, but this time suppose you are trying to construct a 5-by-6 rectangle with the specified properties. Either provide a construction or show that no such exists.

4. BEETLES ON A BOARD

Consider a nine-by-nine chessboard. Suppose a beetle sits at each square of the board. At a given signal, each beetle moves diagonally to a new square on the board. Afterwards some squares will be empty, and others will contain multiple beetles. What is the *minimum* number of unoccupied squares that remain, no matter how the individual beetles move.