Exercises 1

Compute the decimal equivalent of the following

a) $(101101)_2$

b) $(101101)_3$

c) $(AEF)_{16}$

Compute the ternary (base 3) expansion of $(183)_{10}$

Convert from binary to octal to hexadecimal without ever converting to decimal

$(101101011111)_2$

A computer program is a text file that contains a list of instructions for a computer to execute. Assuming computer programs are created using only the symbols found on a standard computer keyboard, find an injective mapping from the set of computer programs to the set of natural numbers.

Is the set of computer programs countable?

Let A be a set containing n elements and let S be the set of all subsets of A (Note that the empty set is considered a subset). Describe a natural bijective mapping between the set S and the set of all n-bit binary numbers. Use this map to give a quick proof that the cardinality of S is 2^n.

Is the set of all subsets of the natural numbers countable or uncountable?

A Weights Problem: A balance scale has a left pan and a right pan. Objects can be placed in each pan and we can determine whether the total weight is the same on each side. We would like to be able to check that certain objects with an advertised integer weight actually have that weight. Suppose that we are allowed to choose five objects of any (positive) integer weight. Using these five objects, we would like to be able to verify all integer weights from 1...n for some maximal n.

Prove that $n \le (3^5 - 1)/2 = 121$.

Is it actually to choose weights that will allow us to achieve this theoretical maximum? If so, how should we choose them? Given an object of weight $n \le 121$, describe an algorithm for deciding how to place our known weights on the scale to determine whether or not the object actually has weight n.