

## Exercises 2

We've seen that the set of operations  $\{+, \bullet, -\}$  are sufficient to represent any boolean function. Using DeMorgan's laws, prove that we actually only need  $\{\bullet, -\}$  or  $\{+, -\}$  to represent an arbitrary boolean function. Is the same thing true for the set  $\{+, \bullet\}$ ?

Prove that any boolean circuit can actually be constructed using only NAND gates.

In this exercise, you will construct a circuit called a **full-adder**. Unlike the half-adder, a full-adder has three boolean inputs that we will call  **$x$ ,  $y$ ,  $c_1$**  that are to be added and two outputs representing the answer called the *sum* and the *carry*. Here the *sum* is the lower order bit and the *carry* is the higher order bit (i.e. the amount that carries over). Write down a truth table for the full-adder and then construct its circuit. You may use any of the gates we have discussed as well as the half-adder itself.