BRAIN TEASERS

1. The Fibonacci sequence $1, 1, 2, 3, 5, 8, 13, \ldots$ starts with two 1's , and each term afterwards is the sum of its two predecessors. Which one of the ten digits is the last to appear in the units position of a number in the Fibonacci sequence?

   (A) 0 (B) 4 (C) 6 (D) 7 (E) 9

2. Consider the sequence

$$1, -2, 3, -4, 5, -6, \ldots$$

   What is the average of the first 200 terms of the sequence?

   (A) -1 (B) -0.5 (C) 0 (D) 0.5 (E) 1

3. Find the sum of all prime numbers between 1 and 100 that are simultaneously 1 modulo 4 and -1 modulo 5.

   (A) 118 (B) 147 (C) 158 (D) 187 (E) 245

4. Prove that the following polynomial has no integral roots:

$$x^8 - 16x^7 + 4x^6 + 13x^5 - 7x^4 + 9x^3 - 2x^2 - 4x + 9$$

5. In the magic square below, the sums of the numbers in each row, column and diagonal are the same. Find $y + z$.

v	24	w
18	x	y
25	z	21

   (A) 43 (B) 44 (C) 45 (D) 46 (E) 47

6. Find the least positive integer n such that no matter how 10^n is expressed as the product of two positive integers, at least one of these two integers contains the digit 0.

7. A positive integer is a *palindrome* if it can be read equivalently from right to left or from left to right (e.g. 505). The year 1991 is the only year in the last century that has the following two properties:it is palindrome, and it factors as a product of a 2 digit prime palindrome and a 3 digit prime palindrome. How many years in the last millennium (1000 to 2000) had the same two properties?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

BRAIN SQUEEZERS

1. Given $0 \leq x_0 < 1$, let

$$x_n = 2x_{n-1}$$

if $2x_{n-1} < 1$,

$$x_n = 2x_{n-1} - 1$$

if $2x_{n-1} \geq 1$.

For how many x_0 is it true that $x_0 = x_5$?

(A) 0 (B) 1 (C) 5 (D) 31 (E) $infinitely\ many$

2. If x, y and z are positive numbers satisfying

$$x + \frac{1}{y} = 4$$

$$y + \frac{1}{z} = 1$$

$$z + \frac{1}{x} = \frac{7}{3}$$

then $xyz=$

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{7}{3}$

3. There exist positive integers A, B and C, with no common factor greater than 1, such that

$$Alog_{200}5 + Blog_{200}2 = C.$$

What is $A + B + C$?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

4. Find the sum of all positive integers n for which $n^2 - 19n + 99$ is a perfect square.

5. If x and y are nonzero real numbers such that

$$\mid x \mid + y = 3$$

and

$$\mid x \mid y + x^3 = 0$$

then the integer nearest to $x - y$ is

(A) -3 (B) -1 (C) 2 (D) 3 (E) 5

6. If $x, y > 0$, $log_y x + log_x y = 10/3$ and $xy = 144$, then

$$\frac{x+y}{2} =$$

(A) $12\sqrt{2}$ (B) $13\sqrt{3}$ (C) 24 (D) 30 (E) 36

BRAIN SQUISHERS

1. Prove that the equation

$$x^4 + y^4 + z^4 - 2y^2z^2 - 2z^2x^2 - 2x^2y^2 = 24$$

has no integral solutions.

2. Let $f(x)$ be a continuous function on $[0, a]$, such that $f(x) + f(a - x)$ does not vanish on $[0, a]$. Evaluate the integral

$$\int_0^a \frac{f(x)}{f(x) + f(a-x)} dx.$$