Reconstructing Affine Transformations from Pictures, and Vice Versa

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You can use this short handout as a guideline in reconstructing the formula of an affine transformation from picture, or reconstructing a picture from the formula for an affine transformation.

Suppose you're given the formula for an affine transformation of the form

$$T(\begin{bmatrix} x \\ y \end{bmatrix}) = x\begin{bmatrix} a \\ b \end{bmatrix} + y\begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}.$$

The image of the unit square $\square$ under T will be a parallelogram, with corners

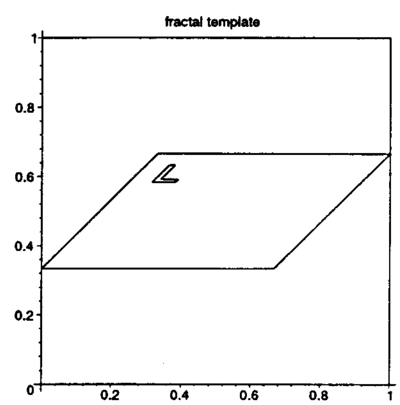
$$T(\begin{bmatrix} 0 \\ 0 \end{bmatrix}) = \begin{bmatrix} e \\ f \end{bmatrix} \quad T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} a+e \\ b+f \end{bmatrix}$$

$$T(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = \begin{bmatrix} a+c+e \\ b+d+f \end{bmatrix} \quad T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} c+e \\ d+f \end{bmatrix}$$

We also need to label the corners; there are at least two ways to do this. First, we can just label each corner as $T(P)$, where P is the appropriate point. Or we can write an "L" inside $\square$, with the long side of the "L" along the left hand side and the short side of the "L" along the bottom side. We then label the parallelogram by the image of this "L" under the affine transformation. Here is an example, which describes the transformation

$$T(\begin{bmatrix} x \\ y \end{bmatrix}) = x\begin{bmatrix} 2/3 \\ 0 \end{bmatrix} + y\begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/3 \end{bmatrix}$$

with a parallelogram labeled by the appropriate "L".



Now suppose you're given a picture of the image of $\square$ under some affine transformation T, with the corners labeled in some way. Let's say the corners are

$$T(\begin{bmatrix} 0 \\ 0 \end{bmatrix}) = A \quad T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = B \quad T(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = C \quad T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = D.$$

Then we can the affine transformation is given by the formula

$$T(\begin{bmatrix} x \\ y \end{bmatrix}) = x(B - A) + y(D - A) + A.$$