1210-90 Exam 2

Spring 2013

Name KEY

Instructions. Show all work and include appropriate explanations when necessary. A correct answer unaccompanied by work may not receive full credit. Please try to do all work in the space provided. Please circle your final answers.

1. (24pts) For this problem, consider the function

$$f(x) = \frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2 + 1.$$

(a) (4pts) Find $f'(x)$.

4

$f'(x) = x^3 - 6x^2 + 9x$

(b) (4pts) Find the critical point(s) of $f(x)$.

4

$0 = f'(x) = x^3 - 6x^2 + 9x = x(x^2 - 6x + 9) = x(x-3)^2$

cps: $x = 0, 3$

2

(c) (2pts) Fill in the blanks: $f(x)$ is decreasing on the interval ($-\infty$, 0). **Note:** $\pm\infty$ are acceptable answers.

(d) (2pts) Which of the critical points you found above is a local minimum?

2

sign f': $-$ on $(-\infty, 0)$, $+$ on $(0, 3)$, $+$ on $(3, \infty)$

$x = 0$ is a local min.

(e) (4pts) Find $f''(x)$.

4

$f''(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3)$

(f) (4pts) Find the inflection point(s) of $f(x)$.

4

$0 = f''(x) = 3(x-3)(x-1)$

sign of f'': $+$ on $(-\infty, 1)$, $-$ on $(1, 3)$, $+$ on $(3, \infty)$

$x = 1, 3$ are inflection pts (or $(1, \frac{15}{4})$ and $(3, {}^{31}/_{4})$)

2

(g) (2pts) Fill in the blanks: $f(x)$ is concave down on the interval (1 , 3). **Note:** $\pm\infty$ are acceptable answers.

(h) (2pts) What is the global minimum value of $f(x)$? If no such minimum exists, write 'DNE'.

2

$f(0) = 1.$

24

2. (12pts) Consider the function

$$f(x) = \frac{x^2+8}{x-1}.$$

(a) (6pts) Find the critical point(s) of $f(x)$ in the interval $[2,5]$.

$$f'(x) = \frac{(x-1)(2x)-(x^2+8)(1)}{(x-1)^2} = \frac{2x^2-2x-x^2-8}{(x-1)^2} = \frac{x^2-2x-8}{(x-1)^2}.$$ 3

6

$0 = f'(x) \Rightarrow 0 = x^2-2x-8 = (x-4)(x+2)$ 2

$x=4$ is only cp in $[2,5]$ 1

(b) (6pts) Find the extreme values of $f(x)$ on the interval $[2,5]$. Max= 12 Min= 8

Check cps in $(2,5)$ and check endpoints $x=2$ and $x=5$

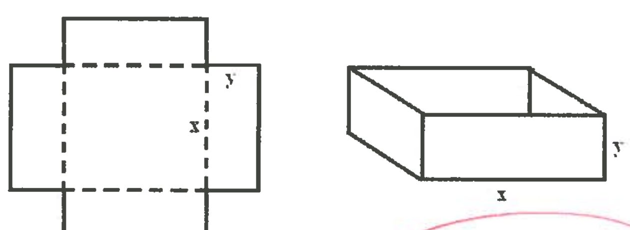
6

$f(4) = \frac{24}{3} = 8$

$f(2) = \frac{12}{1} = 12$

$f(5) = \frac{33}{4} = 8\frac{1}{4}$

3. (12pts) A box with a square base and no top is to be constructed out of 300 in^2 of cardboard. The picture on the left shows the piece of cardboard (which has area 300 in^2), and the picture on the right shows the box obtained by folding along the dotted lines. What are the dimensions of the box (labeled x and y in the pictures below) that will maximize the volume of the box? **Note:** You must use calculus to get credit!!



12

Want to maximize $V = x^2y$ 3 subject to the constraint

$300 = A = x^2 + 4xy$ 3 $\Rightarrow y = \frac{300-x^2}{4x}$

So

$$V(x) = x^2\left(\frac{300-x^2}{4x}\right) = 75x - \frac{1}{4}x^3$$

$0 = V'(x) = 75 - \frac{3}{4}x^2$ 3 $\Rightarrow x^2 = 100 \Rightarrow x = 10.$

Note: sign V': $+$ before 10, $-$ after 10 $\Rightarrow$ $x=10$ is max. 3

So $x = 10$, $y = 5$

24

4. (10pts) Consider the function $f(x) = 3x^2 + 1$ on the interval $[0, 2]$.

(a) (5pts) Find the value of c guaranteed by the Mean Value Theorem.

5

$6c = f'(c) = \frac{f(2)-f(0)}{2-0} = \frac{13-1}{2} = 6 \Rightarrow c = 1$

(b) (5pts) Find the value of c guaranteed by the Mean Value Theorem for Integrals.

5

$3c^2+1 = f(c) = \frac{1}{2-0}\int_0^2 (3x^2+1)\,dx = \frac{1}{2}\left(x^3+x\right|_0^2 = 5$

So $3c^2 = 4 \Rightarrow c = \sqrt{\frac{4}{3}}$

5. (16pts) Find the indicated antiderivatives:

−2 total for forgetting +C

(a) (4pts) $\int (x+9)\,dx$

4

$\frac{x^2}{2} + 9x + C$

(b) (4pts) $\int \sqrt{x}\,dx = \int x^{1/2}\,dx$

4

$\frac{2}{3}x^{3/2} + C$

(c) (4pts) $\int (x^4+x)^7(4x^3+1)\,dx = \int u^7\,du = \frac{1}{8}u^8 + C = \frac{1}{8}(x^4+x)^8 + C$

4

$u = x^4 + x$

$du = (4x^3+1)\,dx$

(d) (4pts) $\int \sin(5x-3)\,dx = \frac{1}{5}\int \sin u\,du = -\frac{1}{5}\cos u + C$

4

$u = 5x-3$

$du = 5dx \Rightarrow dx = \frac{1}{5}du$

$= -\frac{1}{5}\cos(5x-3) + C$

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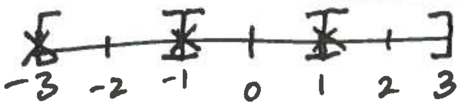
6. (a) (5pts) Approximate $\int_{-3}^{3} \sqrt{9-x^2}\,dx$ using a Riemann sum with 3 subintervals of equal length the sample points being the left-endpoints of the subintervals.

5



$\Delta x = \frac{3-(-3)}{3} = 2.$

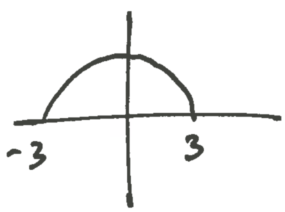
$f(x) = \sqrt{9-x^2}$

$= f(-3)\Delta x + f(-1)\Delta x + f(1)\Delta x = 0(2) + \sqrt{8}(2) + \sqrt{8}(2)$

$= 4\sqrt{8} = 8\sqrt{2}$

(b) (5pts) Find the exact value of $\int_{-3}^{3} \sqrt{9-x^2}\,dx$. **Hint:** Use geometry and the area interpretation of the definite integral. What does the graph of $y = \sqrt{9-x^2}$ look like?

5



graph of $y = \sqrt{9-x^2}$ between $x=-3$ and $x=3$ is top half of circle of radius 3. So

$\int_{-3}^{3} \sqrt{9-x^2}\,dx$ = area under graph $= \frac{1}{2}(\pi(3)^2) = \frac{9}{2}\pi$

7. (10pts) Evaluate the indicated definite integrals:

(a) (5pts) $\int_0^2 (4x^3 - 2x)\,dx$

5

$= \left(x^4 - x^2\right|_0^2 = (16-4) - (0-0) = 12$

(b) (5pts) $\int_0^1 x\sqrt{x^2+1}\,dx$ $= \frac{1}{2}\int_1^2 u^{1/2}\,du = \frac{1}{2}\left(\frac{2}{3}u^{3/2}\right|_1^2$

5

$u = x^2+1$

$du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}du$

$= \frac{1}{3}(2^{3/2} - 1) = \frac{1}{3}(\sqrt{8}-1)$

8. (6pts) The velocity of a particle moving along the x-axis is given by $v(t) = t^2 - 4t + 3$. Assume that the units of the axis measure meters and the velocity is given in meters per second. If the particle is located at $x = 5$ at time $t = 0$, where is the particle located at time $t = 3$?

$s(t)$ = position of particle at time t. $s'(t) = v(t)$ 2

6

$s(3) = s(0) + \int_0^3 s'(t)\,dt$ 2

$= 5 + \int_0^3 (t^2 - 4t + 3)\,dt = 5 + \left(\frac{1}{3}t^3 - 2t^2 + 3t\right|_0^3$

$= 5 + (9 - 18 + 9) = 5$ 2

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