Calculus I
Exam 1, Summer 2002, Answers

1. Find the equation of the line which goes through the point (2,-1) and is parallel to the line given by the equation $2x+3y=10$.

Answer. To find the slope, write the equation as $y=(-2x+10)/3$. Then the slope of any line parallel to the given line is -2/3. The line we seek goes through (2,-1), so its equation is

$$\frac{y+1}{x-2}=-\frac{2}{3}$$

which simplifies to $y=-(2/3)x+1/3$ or $2x+3y=1$.

Alternatively, we note that the equation of the general line parallel to the given line must be of the form $2x+3y=C$, for some C. Substitute $x=2,\ y=-1$ to find $C=1$.

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2. Find the derivatives of the following functions:

a) $f(x)=2x^3-8x^2+1$

Answer. $f'(x)=6x^2-16x$.

b) $g(x)=(x^2+1)(x^{-2}+1)$

Answer. By the product rule,

$$g'(x)=(x^2+1)(-2x^{-3})+(2x)(x^{-2}+1)=-2x^{-1}-2x^{-3}+2x^{-1}+2x=-2x^{-3}+2x$$

Alternatively, avoid the product rule bu multiplying out: $g(x)=1+x^2+x^{-2}+1$, and now differentiate.

c) $h(x)=\dfrac{x+1}{x^2+1}$

$$h'(x)=\frac{x^2+1-(x+1)(2x)}{(x^2+1)^2}=\frac{-x^2-2x+1}{(x^2+1)^2}\ .$$

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3. Find the derivatives of the following functions:

a) $f(x)=(\cos(2x)+1)^3$

Answer. $f'(x)=3(\cos(2x)+1)^2(-\sin(2x))(2)=-6\sin(2x)(\cos(2x)+1)^2$.

b) $g(x)=(2x+1)^{-1}$

Answer. $g'(x)=-(2x+1)^{-2}(2)=-2/(2x+1)^{-2}$.

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4. Find the equation of the line tangent to the curve $y=(x^2-1)^2$ at (2,9).

Answer. To find the slope of a tangent line, we differentiate: $y' = 2(x^2-1)(2x) = 4x(x^2-1)$. At $x = 2$, we get $m = 4(2)(4-1) - 24$. The equation then is

$$\frac{y-9}{x-2} = 24$$

which simplifies to $y = 24x - 39$.

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5. An object moves in a straight line so that its position at time t is given by $x(t) = (\sin t)^2$. What is the velocity of the object when $t = 3\pi/4$?

Answer. $v(t) = x'(t) = 2(\sin t)(\cos t)$. Evaluating at $t = 3\pi/4$:

$$v = 2(\frac{\sqrt{2}}{2})(-\frac{\sqrt{2}}{2}) = -1 \ .$$