

**Calculus I**  
**Final Exam, Fall 2002**

1. Find the derivatives of the following functions:

a)  $f(x) = (x^2 - 3)^2(2x + 4)$

b)  $g(x) = \tan x \sin x$

2. Integrate:

a)  $\int (6x^2 + 1)^5 x dx$

b)  $\int \frac{dy}{y^{5/2}}$

3. Find the point in the first quadrant at which the tangent line to the curve  $2y^2 + 6x - y = 3$  has slope equal to 3.

4. Consider the region in the first quadrant bounded by the curve  $y = 12 - \frac{3}{4}x^2$ . What are the dimensions of the largest rectangle with sides parallel to the coordinate axes which can be inscribed inside this region?

5. Sketch the graph of the function  $y = 3x^4 - 4x^3 - 12x^2 + 2$ . Find the  $x$  values of all local minima, maxima and points of inflection.

6. Solve the initial value problem:  $\frac{dy}{dx} = y^2x$        $y(0) = 2$

7. Find the area between the curve

$$y = \frac{x+1}{x^3}$$

and the  $x$ -axis, as  $x$  ranges from 1 to 4.

8. Find the volume of the solid obtained by rotating about the  $y$ -axis the region bounded by the curves  $y = 4x$ ,  $y = 5 - x^2$ ,  $y = 0$ .

9. Consider a curve given parametrically by  $x = 4 \cos^2 t$ ,  $y = 3 \sin^2 t$ . Find the length of the piece of this curve running from  $t = 0$  to  $t = \pi/2$ .

10. The square with vertices at  $(0,0)$ ,  $(1,0)$ ,  $(1,1)$ ,  $(0,1)$  is filled with a material whose density at the point  $(x,y)$  is  $\delta(x,y) = x(1-x)$  g/cm<sup>2</sup>. What is the mass of this object? What is its moment about the  $y$ -axis?