

# MATH 1210-90 Fall 2011

## Final Exam

INSTRUCTOR: H.-PING HUANG

**Hint:** do NOT calculate any numerical value, unless specified otherwise.

LAST NAME \_\_\_\_\_

FIRST NAME \_\_\_\_\_

ID NO. \_\_\_\_\_

**INSTRUCTION:** SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1 30 \_\_\_\_\_

PROBLEM 2 30 \_\_\_\_\_

PROBLEM 3 30 \_\_\_\_\_

PROBLEM 4 30 \_\_\_\_\_

PROBLEM 5 30 \_\_\_\_\_

PROBLEM 6 30 \_\_\_\_\_

PROBLEM 7 30 \_\_\_\_\_

PROBLEM 8 10 \_\_\_\_\_

TOTAL 160 \_\_\_\_\_

## PROBLEM 1

(30 pt) Evaluate the limit

$$\lim_{h \rightarrow 0} \frac{3(5+h)^2 + 3(5+h) - (3 \cdot 5^2 + 3 \cdot 5)}{h}$$

## PROBLEM 2

(30 pt) Find the equation of the tangent line to the curve  $y = (x + 1)(x^2 - 1)$  at the point  $(1, 0)$ .

## PROBLEM 3

(30 pt) The function  $f(x) = 4x^3 + 6x^2 - 72x$  is decreasing on the interval  $(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$ . It is increasing on the interval  $(-\infty, \underline{\hspace{1cm}})$  and the interval  $(\underline{\hspace{1cm}}, \infty)$ .

The function has a local maximum at  $\underline{\hspace{1cm}}$ . Show using both the first and second derivative tests. I expect two separate responses, each with their own work.

## PROBLEM 4

(30 pt) Consider the parametric equation

$$x = \cos \theta + \theta \sin \theta \quad y = \sin \theta - \theta \cos \theta$$

What is the length of the curve for  $\theta = 0$  to  $\theta = 7/2\pi$ ?

## PROBLEM 5

(30 pt) Set up the finite integral for the volume formed by rotating the region inside the first quadrant enclosed by

$$y = x^4 \quad y = 8x$$

(a) about the  $x$ -axis.

(b) about the  $y$ -axis.

Please do not evaluate the values.

## PROBLEM 6

(30 pt) Given

$$f(x) = \int_0^x \frac{t^2 - 1}{1 + \cos^2 t} dt$$

At what value does the local **max** of  $f(x)$  occur?

**Hint:** use the first derivative test .

## PROBLEM 7

(30 pt)

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \frac{6}{n}\right)^2} \cdot \frac{6}{n} + \frac{1}{\left(1 + \frac{12}{n}\right)^2} \cdot \frac{6}{n} + \frac{1}{\left(1 + \frac{18}{n}\right)^2} \cdot \frac{6}{n} + \cdots + \frac{1}{\left(1 + \frac{6n}{n}\right)^2} \cdot \frac{6}{n} \\ = \int_1^b f(x) \, dx. \end{aligned}$$

Find out the upper limit  $b$ , and the integrand  $f(x)$ , and the finite integral  $\int_1^b f(x) \, dx$ .



## PROBLEM 8

(10 pt) Evaluate the definite integral

$$\int_3^5 \left( \frac{d}{dt} \sqrt{4 + 3t^4} \right) dt$$