Solutions for Introduction to Polynomial Calculus

Section 4 Problems - Antiderivatives of Polynomials

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Calling the function in each problem $f(x)$ and using the three antidifferentiation rules corresponding to the previous three differentiation rules:

The antiderivative of $f(x) = x^n$ is $\int f(x)dx = \frac{x^{n+1}}{n+1} + C$.

If $f(x) = u(x) + v(x)$ then $\int f(x)dx = \int u(x)dx + \int v(x)dx$.

If $f(x) = c(u(x))$ where c is a constant, then $\int f(x)dx = c \int u(x)dx$.

(1) $\int f(x)dx = x^2 - 3x + C$. You should check this by taking its derivative!

(2) $\int f(x)dx = x^3 - 2x^2 + 5x + C$.

(3) $\int f(x)dx = \frac{x^6}{6} + \frac{x^4}{2} + x + C$.

(4) $\int f(x)dx = x^{10} - 4x^2 + C$.

Find the general antiderivative then impose the condition to determine C:

(5) $F(x) = \int f(x)dx = \frac{x^3}{3} - 5x + C$ and $F(0) = 2$ says $C = 2$, so $F(x) = \frac{x^3}{3} - 5x + 2$.

(6) $F(x) = \int f(x)dx = 2x^4 - x^2 + C$ and $F(1) = 4$ says $2 - 1 + C = 4$, so $C = 3$ and $F(x) = 2x^4 - x^2 + 3$.

(7) $F(x) = \int f(x)dx = \frac{x^4}{2} + C$ and $F(1) = 1$ says $\frac{1}{2} + C = 1$, so $C = \frac{1}{2}$ and $F(x) = \frac{x^4}{2} + \frac{1}{2}$.

(8) $F(x) = \int f(x)dx = \frac{x^4}{4} - \frac{x^2}{2} + C$ and $F(2) = 1$ says $4 - 2 + C = 1$, so $C = -1$ and $F(x) = \frac{x^4}{4} - \frac{x^2}{2} - 1$.

(9) The derivative of velocity is acceleration, and the acceleration of any body near the earth's surface under only the force of gravity is -32 feet per second squared. Since the (vertical) velocity is then the antiderivative of the acceleration,

$$v(t) = \int a(t)dt = \int -32dt = -32t + C$$

feet per second. We are given that $v(0) = 64$ feet per second, so $0 + C = 64$ and $v(t) = -32t + 64$ feet per second is the velocity after t seconds. The ball will achieve its maximum height when its vertical velocity changes from positive to negative, i.e., when $v(t) = -32t + 64 = 0$, so when $t = 2$ seconds.

(10) The derivative of (vertical) displacement, or height, is velocity, and the velocity of the ball is $v(t) = -32t + 64$ from the previous problem. Since the (vertical) displacement is then the antiderivative of the velocity,

$$s(t) = \int v(t)dt = \int -32t + 64dt = -16t^2 + 64t + C$$

feet. We are given that $s(0) = 6$ feet, so $0 + 0 + C = 6$ and $s(t) = -16t^2 + 64t + 6$ feet is the height of the ball after t seconds. Since the ball achieves its maximum height when $t = 2$ seconds, the maximum height it achieves is $s(2) = 70$ feet.